



Jost Bürgi in Drehung

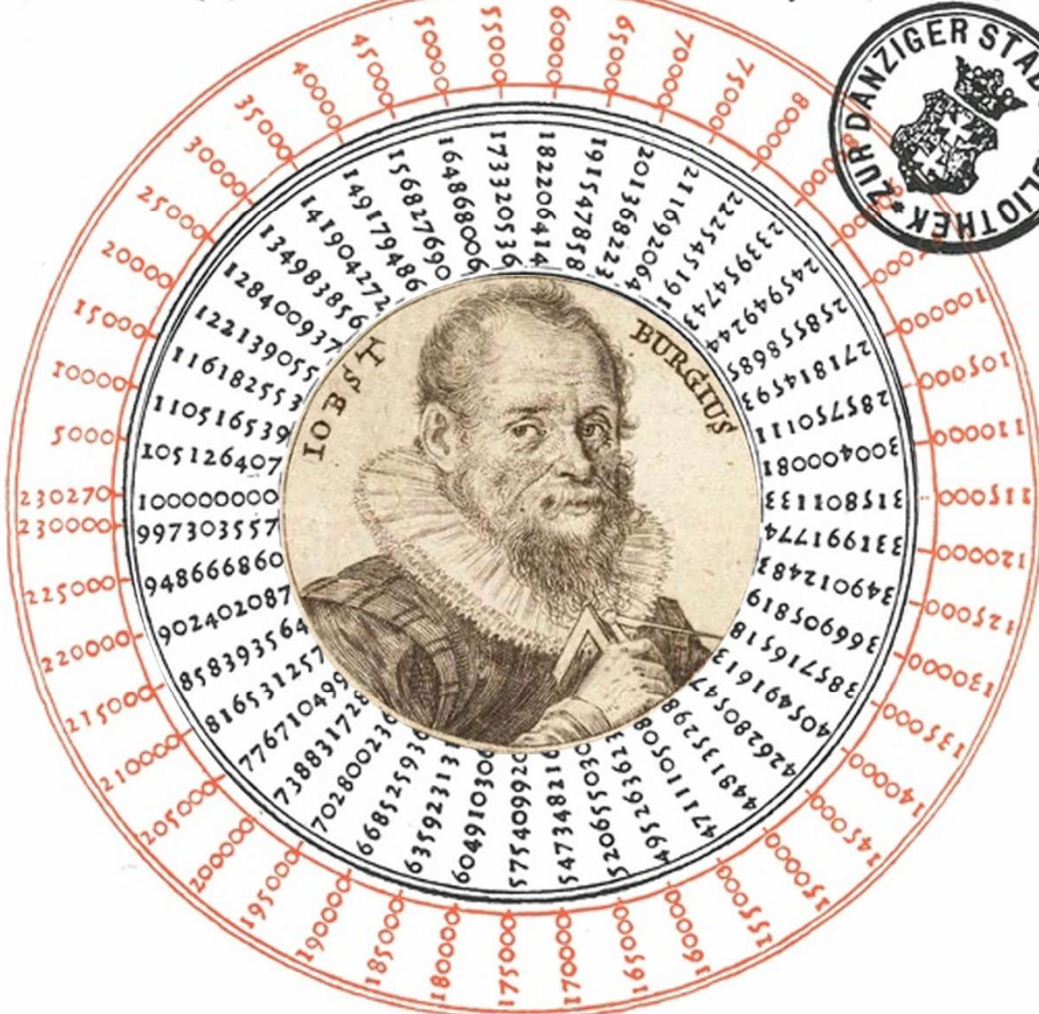
29. April 2022

Daniel Muzzolini
Zürcher Hochschule der Künste
Jost-Bürgi-Symposium 2022

Aritmetische vnd Geometrische Progreß

Tabulen/sambt gründlichem vnterricht/wie solche nützlich
in allerley Rechnungen zugebrauchen/vnd verstanden werden sol.

(Ein - nütz - liche - und - nützlich - ist - ein - Mann - sehr - beizufügen)



	0	500	1000
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30	30003	31380	45271
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80	80028	81656	120799
90	90036	91714	135907
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220	10231	12562	17411
230	10253	12634	17533
240	10276	12707	17657
250	10300	12782	17782
260	10325	12857	17907

Gedruckt/ In der Alten Stadt Prag/ bey Paul

Seßens/der Eöblichen Univerſitet Buchdruckern/ Im Jahr / 1620.



↑ Benjamin Bramer, Bericht ..., 1648

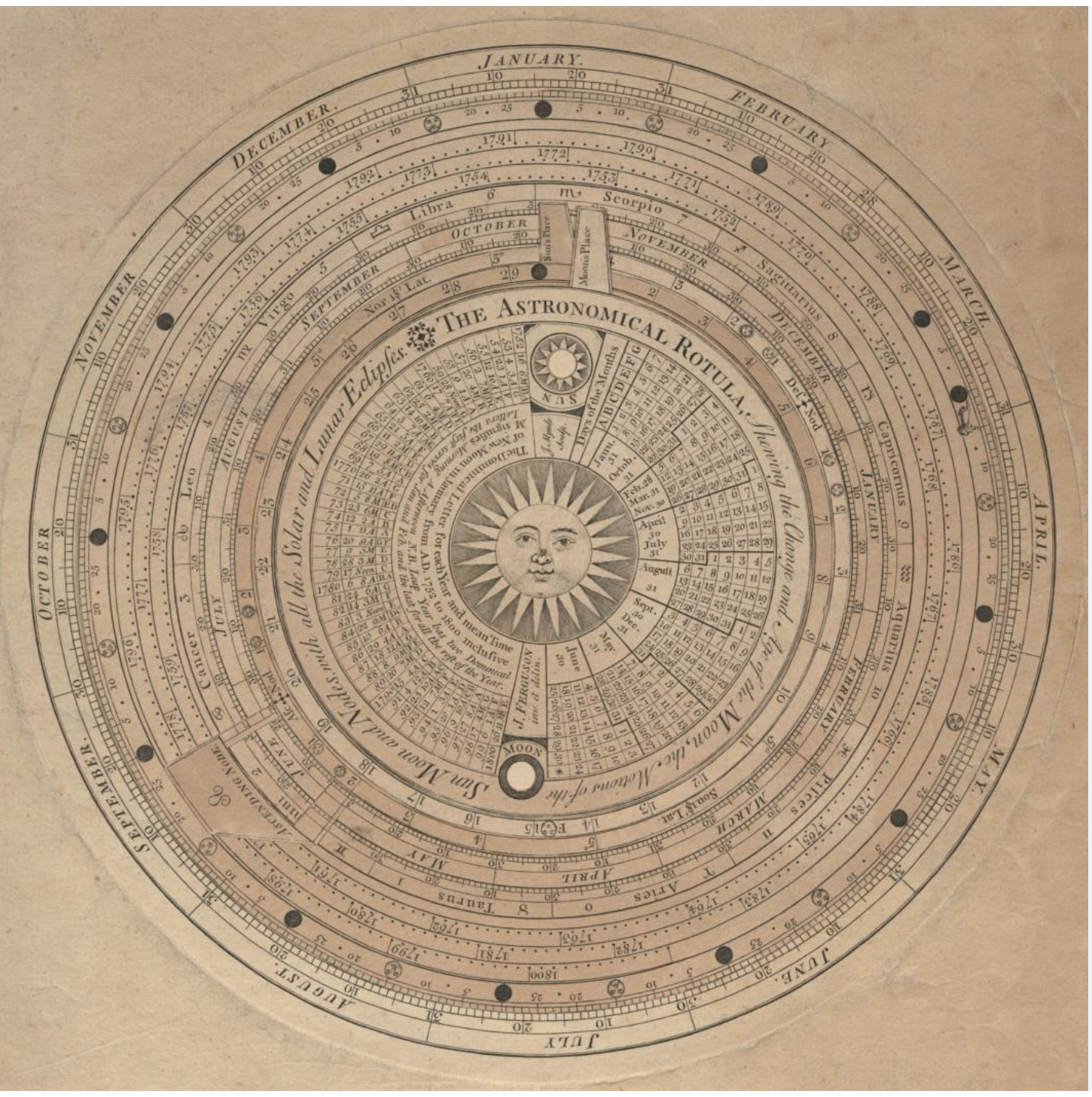
← Salomon de Caus, Institution harmonique, 1615

Small cousins of the Astrolabe

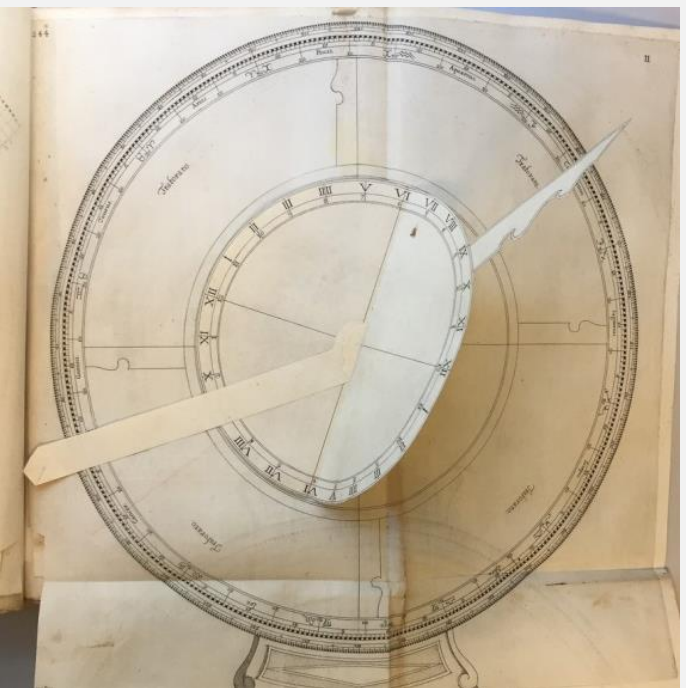
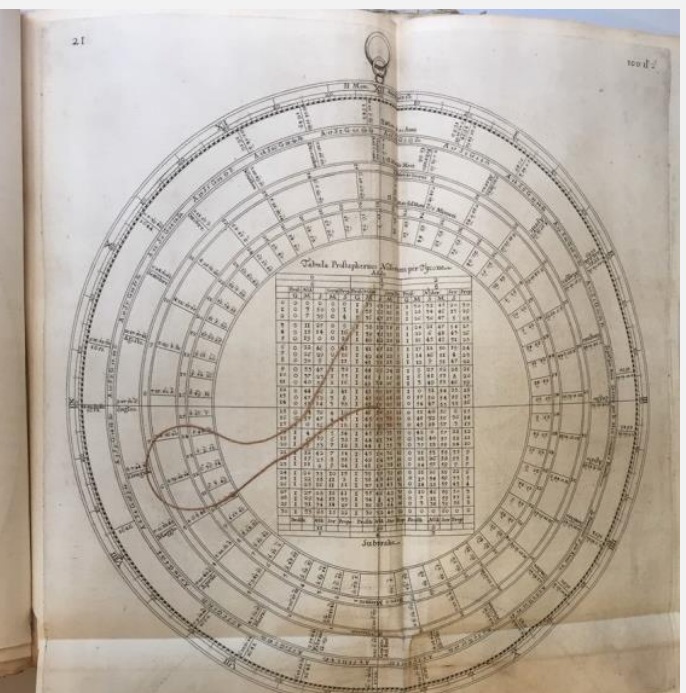
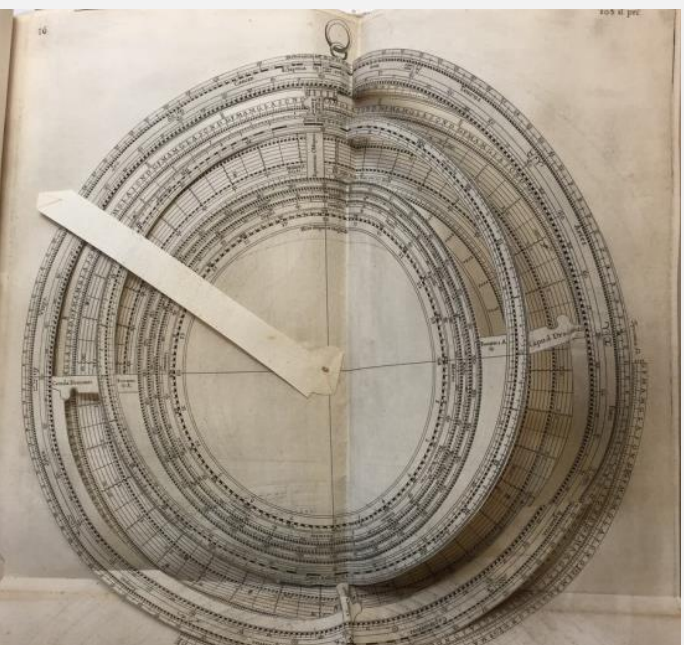
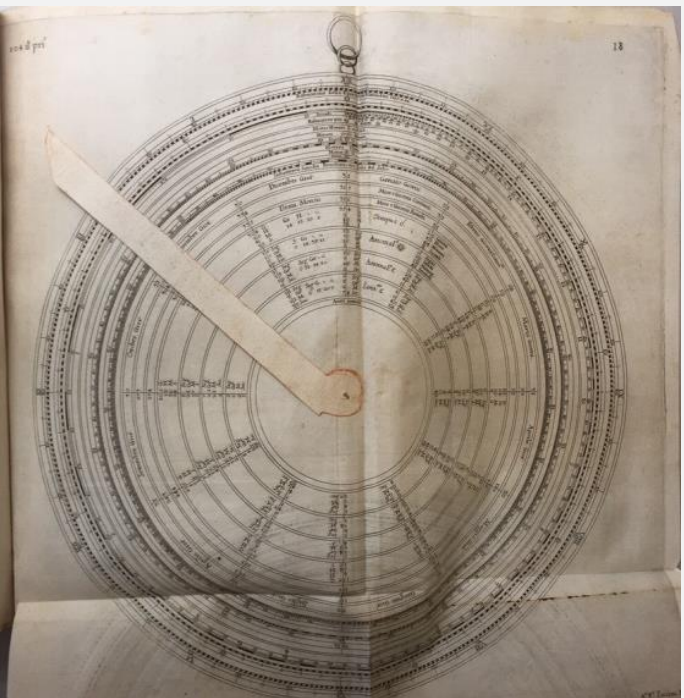
PAPER VOLVELLES

The Astronomical Rotula

James Ferguson, Engraved by J. Mynde,
London



Sir Robert Dudley (1574 – 1649)



Ambrosius Wilflingseder (1563)

CANON.

In superiore & primo circuli spacio inuenies Claves, Scalæ, in sequentibus uero sex circulorum spacijs cuiuslibet clauis uoces, In duobus autem minoris rotæ spacijs quinque Claves signatas, & quatuor finales.

Scala Musica in Plano,

Claves signate	Claves uiginti.	Claves	benolis	Ficta
			fa	fa
				fa
	cc	la		
D	dd	la-sol		
	cc	sol fa	fa	
	bb	fa mi		fa
	aa	la mi re		fa
	g	sol re ut		
	f	fa nt	fa	fa
	e	la mi		fa
	d	la sol re		
	c	Affinales- sol fa ut.	-fa	
	b	quatuor fa mi		fa
	a	la mi re		fa
	G	sol re ut		
	F	Finiales- fa ut	fa	fa
	E	quat: la mi		fa
	D	sol re		
	C	fa ut	fa	
	B	mi		fa
	A	re		fa
F	F	ut	fa	fa

Quare

The second part

we apply the index of the Moone, it shall shewe in the circle the signes, the place where she is. And so shall she appear in the instrument lightened, or darkened, more or lesse as in her nature. In like manner, considering the place of the Sunne in the sphere, shall be scene what aspect they haue, by the lines that traunce the superficiall of the Lunar circle of the Moone.



of the Arte of Navigation.

Fol. 33.

Five aspectes of the planets.

The aspectes which the Planets haue one to another, or where by they beholde one another, are five.

Coniunction, when two Planets be vnder one selfe same degree and minute in the Zodiacke, whose charact is this. δ

Opposition, when betwene the place of the Planettes is halfe a circle, which are. 180. degrees, and is thus figured. ∞

Trinall, when betwene the Planettes shall be foure signes, which are. 120. degrees, and is figured thus. Δ

Quadrine, when one Planet is distant from another by thre signes, which are. 90. degrees, whose charact is this. $\square$

Sextile, when two signes are betwene them which are thre scope degrees, and is marked thus. $\ast$

And yf by memorie you desyre to knowe the true space of the Sunne, without respect of minutes (whiche may sufficiently be done with the Astrolabe) beare in memorie these numbers 10. 9. 10. 11. 12. 13. 14. 13. 14. 13. 12. Of the which the first serueth for Januarie, the seconde for Februarie, with the other signes: and so of the rest. Then to knowe in what degree the Sunne is, you shall take away the dayes that are applyed to every Moneth, according to the sayd numbers of the dayes for the which you desyre to knowe the true place of the Sunne, and in them that remaine, in so many degrees is the Sunne of the signe into the which it entred that Moneth. And yf the dayes past of the Moneth, shall be lesse then the dayes applyed to the same Moneth: you shall ioyne thirtie with those dayes past of the Moneth, and of the summe that amounteth, you shall take away the dayes applyed to the sayd Moneth, and the rest shall be the degrees in which the Sunne shall be of the signe of the Moneth past, as for example,

To know the Place of the Sunne by the rule of memorie.

To know in what degree the Sunne is.

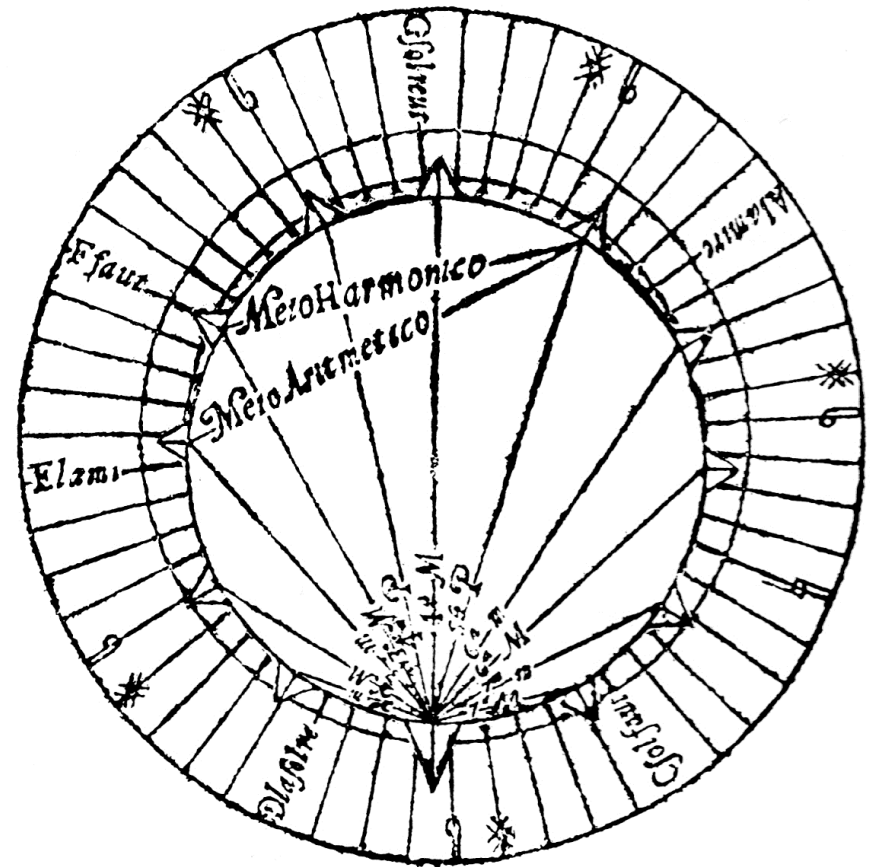
E. 1.

Example.

Astronomy vs Music

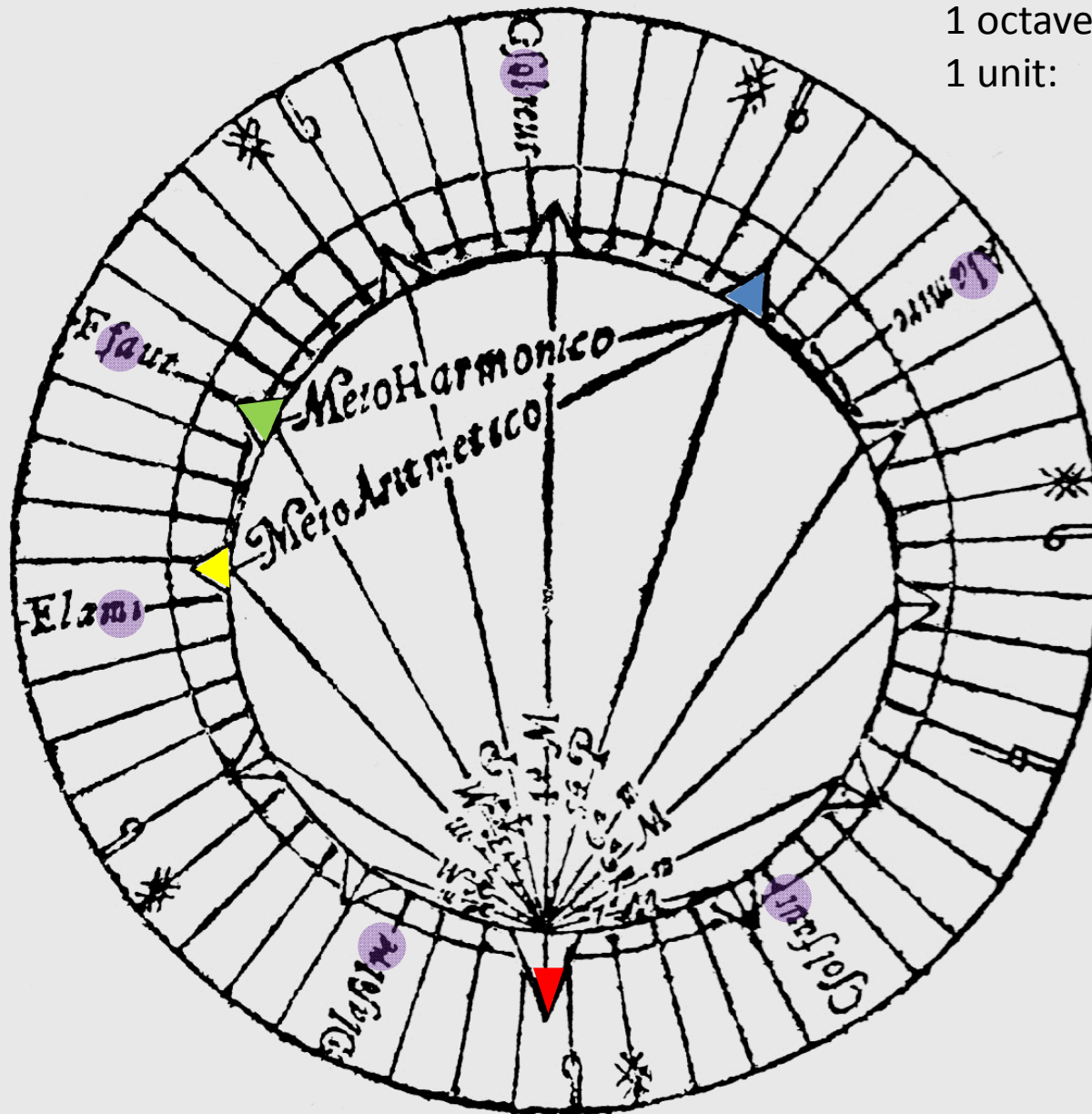


Martín Cortés, *Art of Navigation*, 1584, ff 32v



Antonio Fernández: *Arte de Musica*, fol 46v (Lindley, 1987, 191)

Antonio Fernandez (1626)



1 octave: $O = 360^\circ$

1 unit: $u = 360^\circ/55$

55-EDO:

Equal Division
of the Octave into
55 equal intervals

Tone $T = 9 u$

Semitone $S = 5 u$

Octave $O = 5 T + 2 S = 55 u$

Hexachords (transposable):

ut – re – mi – fa – sol – la

T T S T T

60 : 50 : 48 : 40

C#/D_b e f G#/A_b

Exponential base:

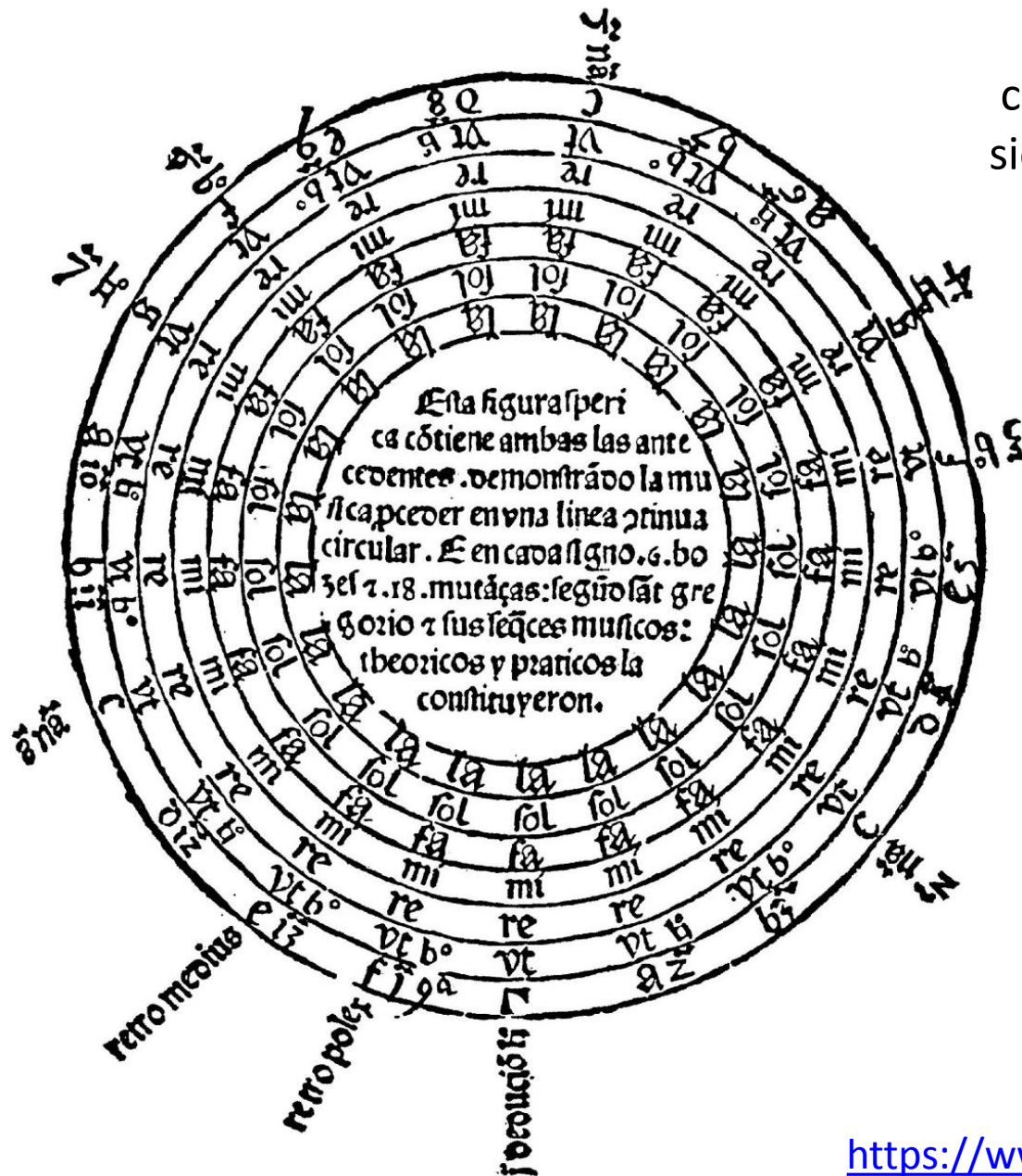
$2^{1/55} = 1.01268$

Why 55?:

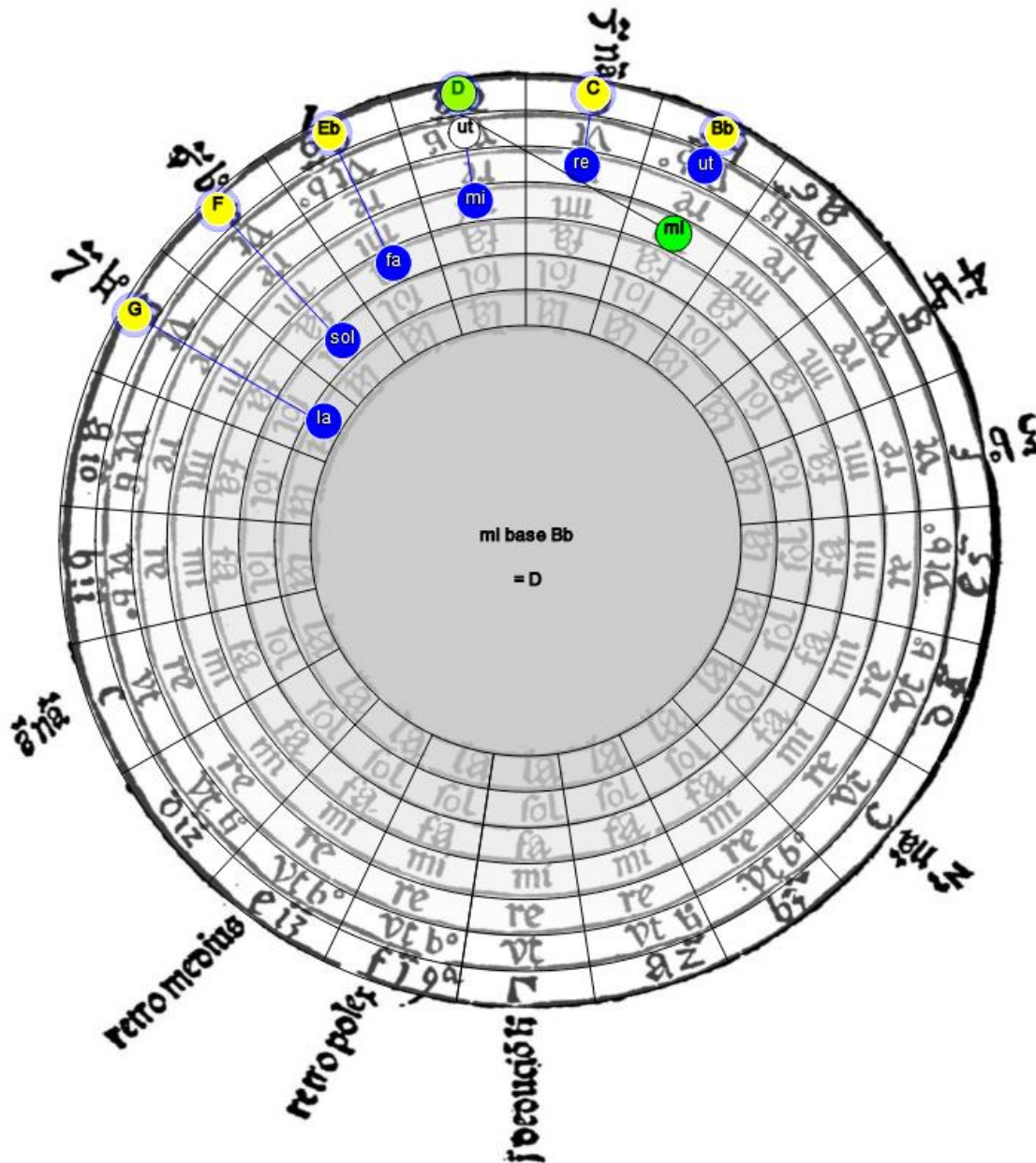
Meantone temperament

Domingo Marcos Durán:
Lux bella (1492)

Esta figura speri
ca cõtiene ambas las ante
cedentes. demonstrãdo la mu
sica pceder en vna linea otinua
circular. [...]



Durán's diagram: a volvelle?



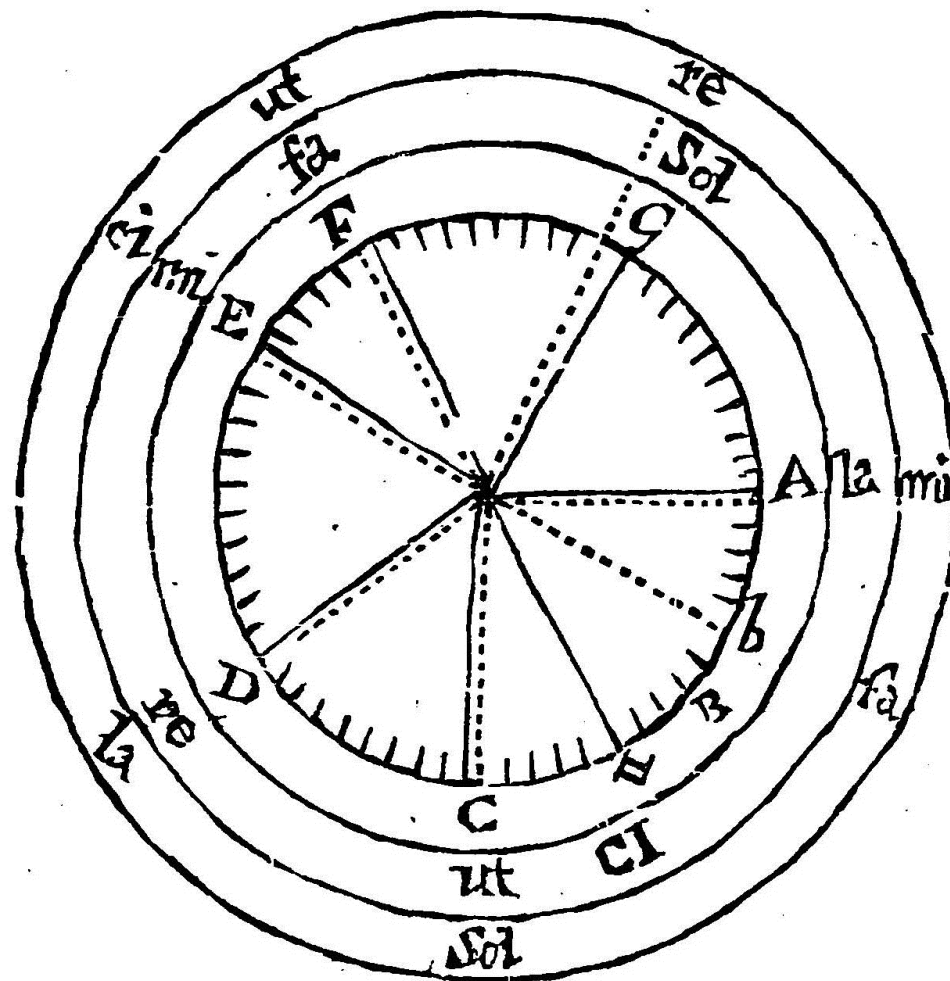
Combinacion de las voces.

Seis sobre agudas.	f					fa	vi		la sol	Conclusa
	c					la	mi	9b	la mi	
	b					la	sol	re	la re	
	c					fa	fa	vi	sol fa	Gregori
	b					fa	mi	sinatura	sol re	
	c								sol vi	
	a					la	mi	re	fa vi	Quinto.
	g					sol	re	vi	mi re	
	f					fa	vi	7b	re vi	
	c					la	mi	9b		Sexto.
siete agu das	b					la	sol	re		
	c					sol	fa	vi		
	b					fa	mi	sin		
	a					la	mi	re		Septimo.
	g					sol	re	vi		
	f					fa	vi	4b		
	c					la	mi	9b		Octavo.
	b					sol	re			
	c					fa	vi			
	b					mi	2 ^a na			Nono.
ocho lras gra ues.	a					la	mi			
	g					sol	re			
	f					fa	vi			
	c					la	mi			Decimo.
	b					sol	re			
	c					fa	vi			
	b					mi	2 ^a na			Undecimo.
	a					la	mi			
	g					sol	re			
	f									Duodecimo.

ro pols	f				la	sol	mi	re
ro naco	c				fol	fa	re	vr
	d				fa	mi	vr	12b
	c				la	mi	re	11x
	b				la	fol	re	vr
	a				fol	fa	vr	10b
	g			la	fa	mi		9x
	f		la	fol	mi	re		
	e		fol	fa	re	vr		
	d		fa	mi	vr	so		
	c		la	mi	re			7x
	b		la	fol	re	vr		
	a		fol	fa	vr	ob		
	g	la	fa	mi				5x
	f	la	fol	mi	re			
	e	fol	fa	re	vr			
	d	fa	mi	vr	4b			
	c	mi	re					
	b	re	vr				la	fol
	a	vr	2b				fol	fa
	conjuncta b					la	fa	mi

1680 Philomus A. B.

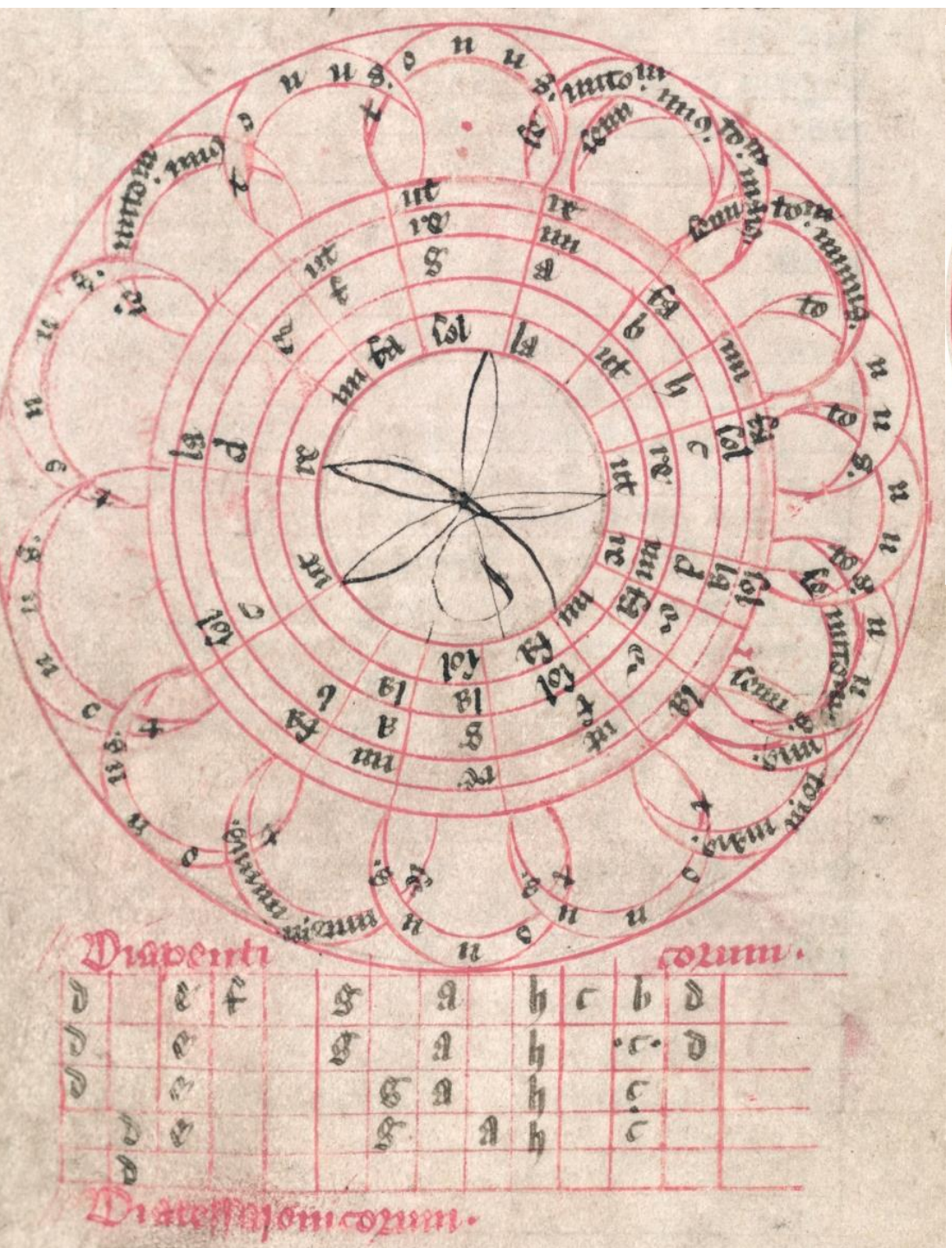
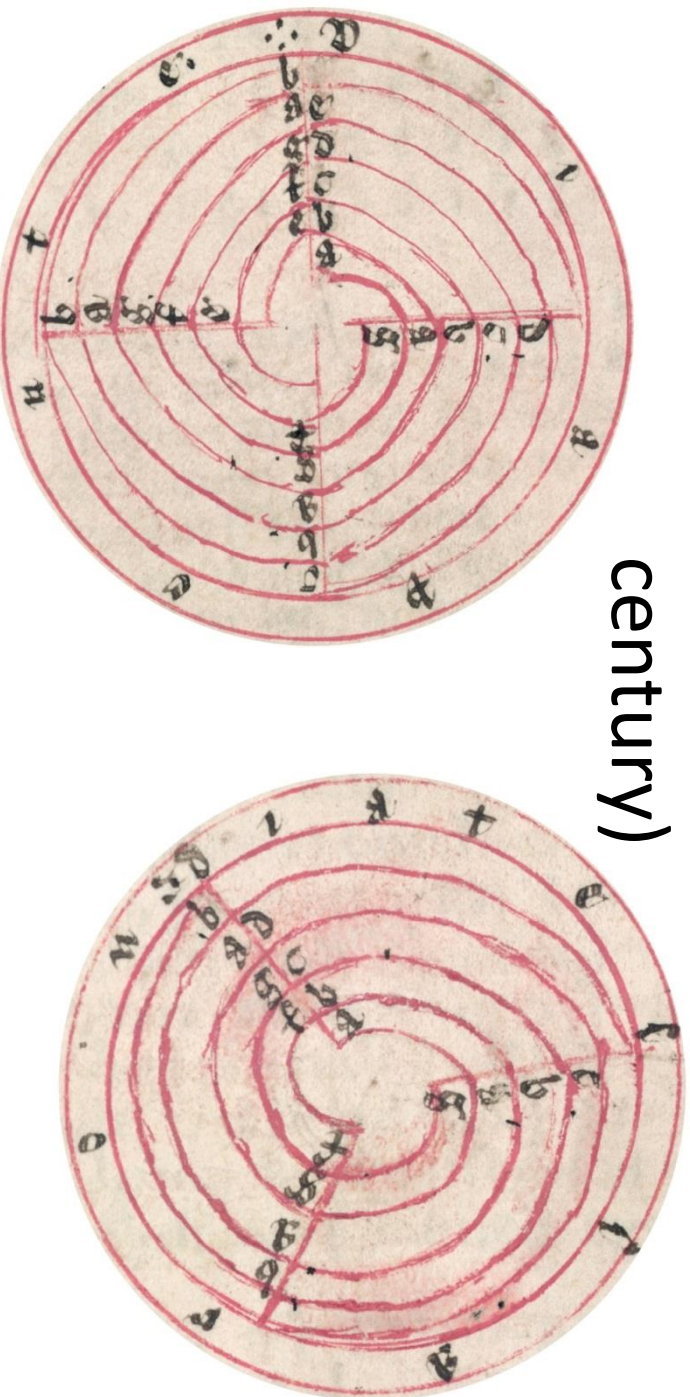
Synopsis of Vocal Musick.



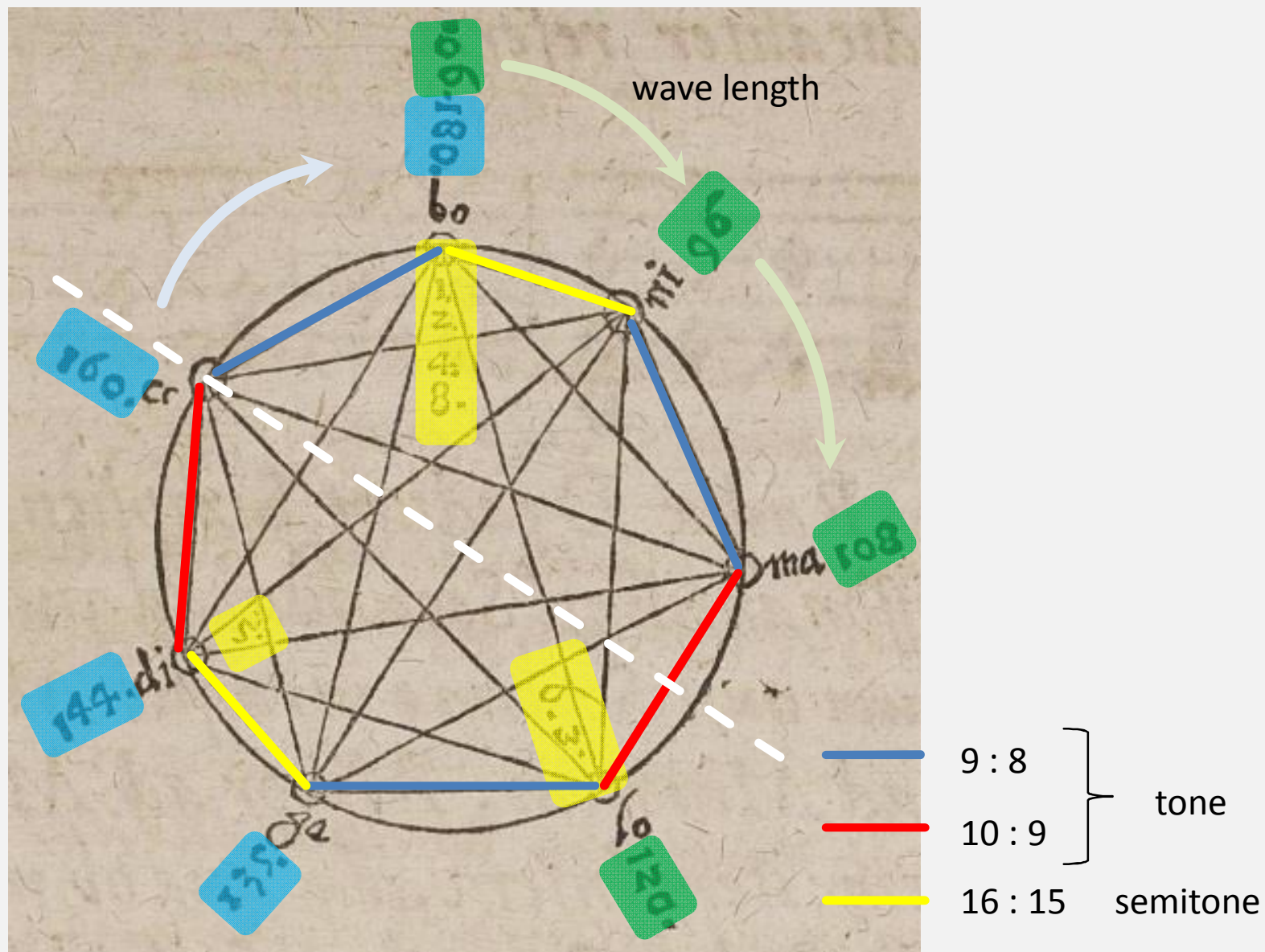
From Theinred to Newton

CIRCULAR PITCH DIAGRAMS

Theinred of Dover (Bodley 842, 12th century)



Johannes Lippius (1612)



René Descartes and Zarlino's Senario

$T = \{1, 2, 3, 4\}$: Pythagorean Tetraktys, perfect consonances pairs from T , as ratios

$S = \{1, 2, 3, 4, 5, 6\}$: Senario

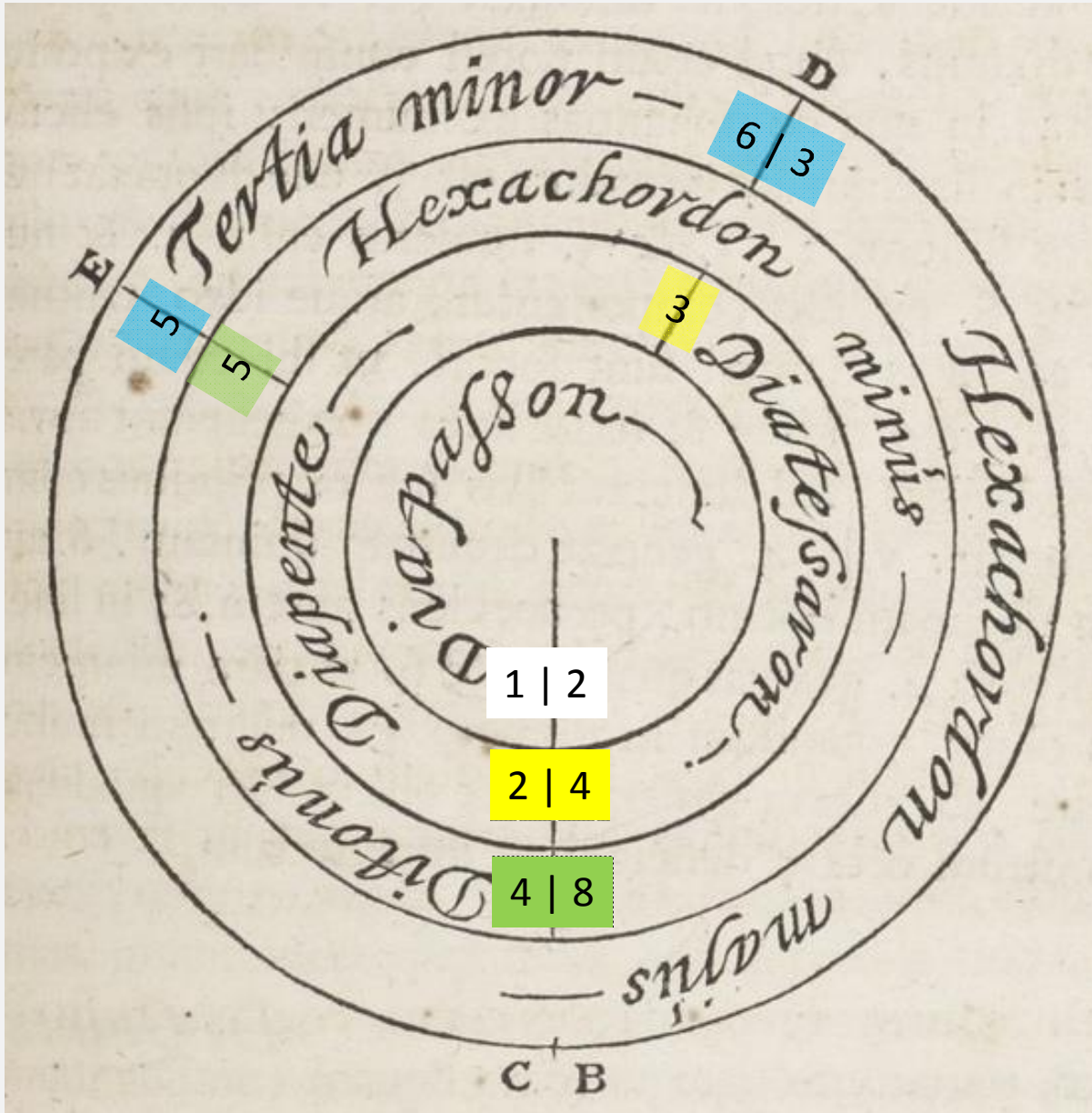
$R = \{s_1/s_2 \mid (s_1, s_2) \text{ in } S \times S, s_1 < s_2\}$ The «syntonic consonances»: consonances of just intonation

$\frac{1}{2}$	Octava.								
$\frac{1}{3}$	Duodecima	$\frac{2}{3}$	Quinta						
$\frac{1}{4}$	Decima 5 ^a .	$\frac{2}{4}$	Octava	$\frac{3}{4}$	Quarta				
$\frac{1}{5}$	Decima 7 ^a .	$\frac{2}{5}$	10 ^a maj.	$\frac{3}{5}$	6 ^a maj.	$\frac{4}{5}$	Ditonus		
$\frac{1}{6}$	Decima 9 ^a .	$\frac{2}{6}$	12 ^a .	$\frac{3}{6}$	Octavo	$\frac{4}{6}$	Quinta	$\frac{5}{6}$	Tertia min.



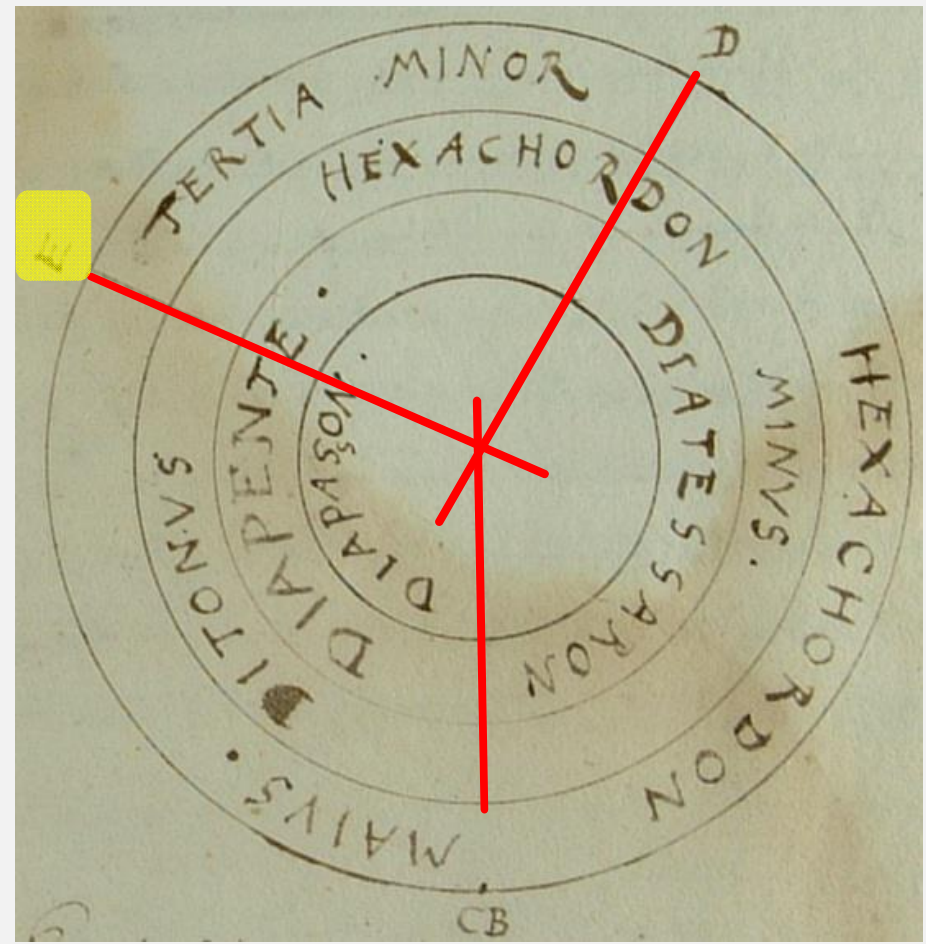
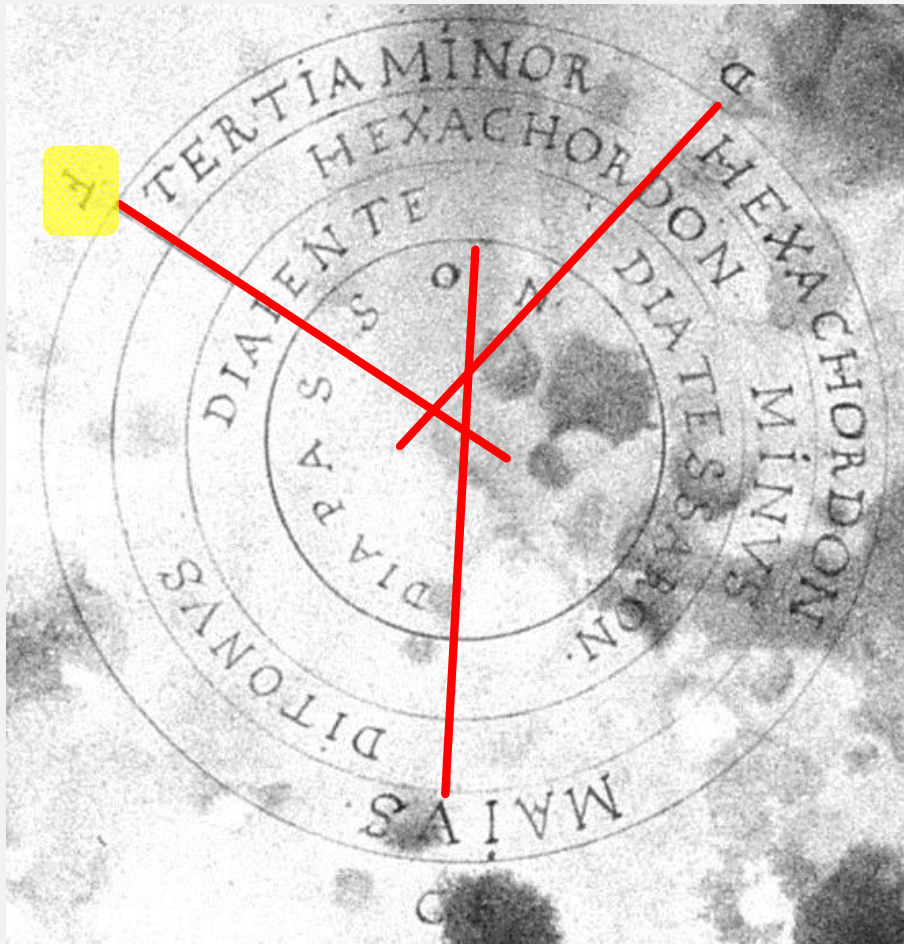
$\{1, 2, 3, 4, 5, 6, 8\}$: 7 excluded!

Descartes (1618/1650)



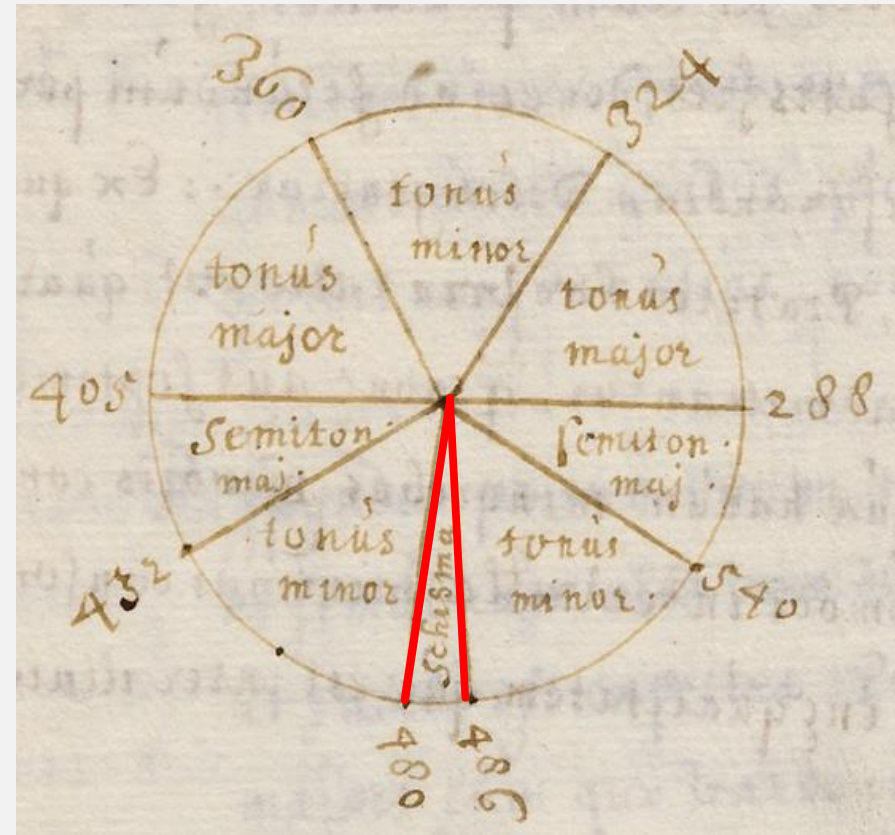
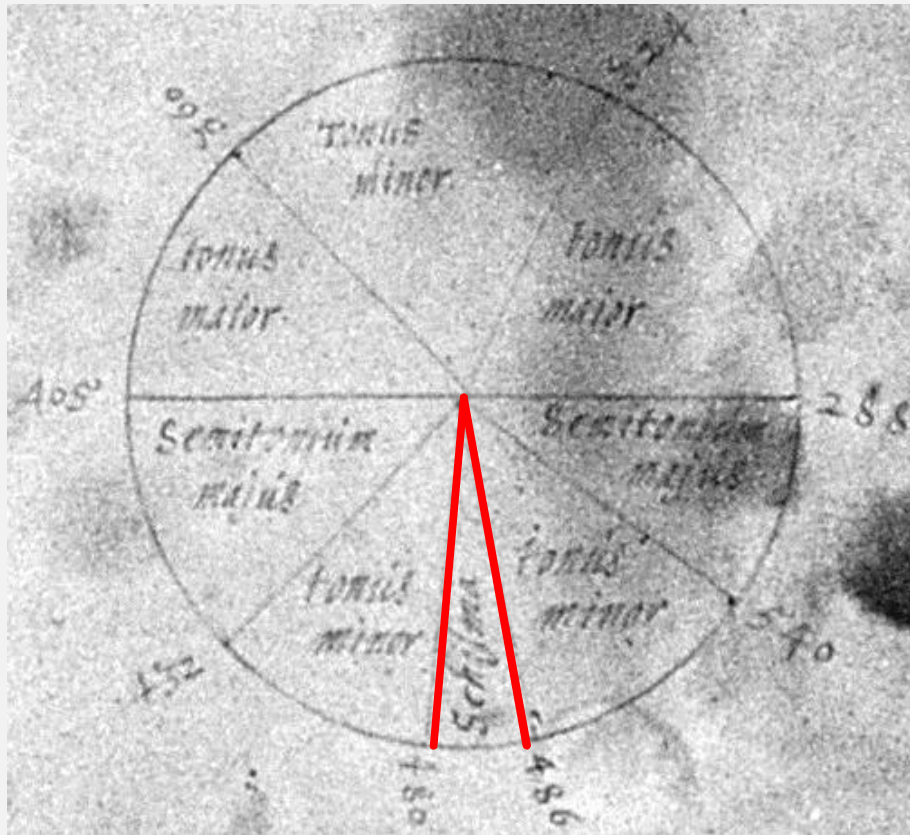
René Descartes, *Compendium musicae* (1618, 1650). The original manuscript (now lost) was a gift for Isaak Beeckman, dated 1.1.1619

René Descartes: Compendium musicae (1619) manuscript copies (c. 1628 / 1635)



<https://sound-colour-space.zhdk.ch/sets/10012> (copy for Beeckman c. 1628)
<https://sound-colour-space.zhdk.ch/sets/10006> Groningen 1640

Descartes Manuscripts (c. 1628 / 1635)

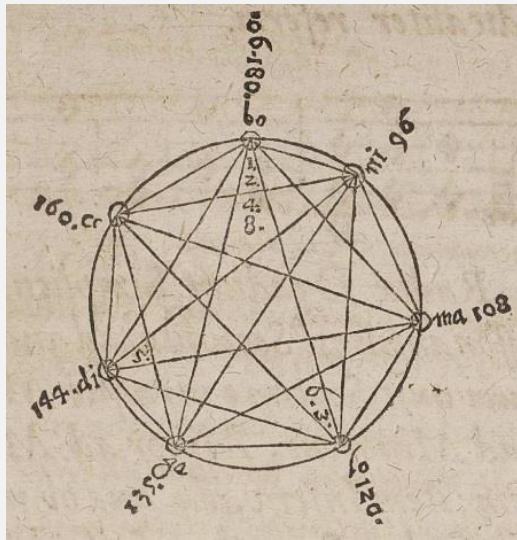


<https://sound-colour-space.zhdk.ch/virtuallab>

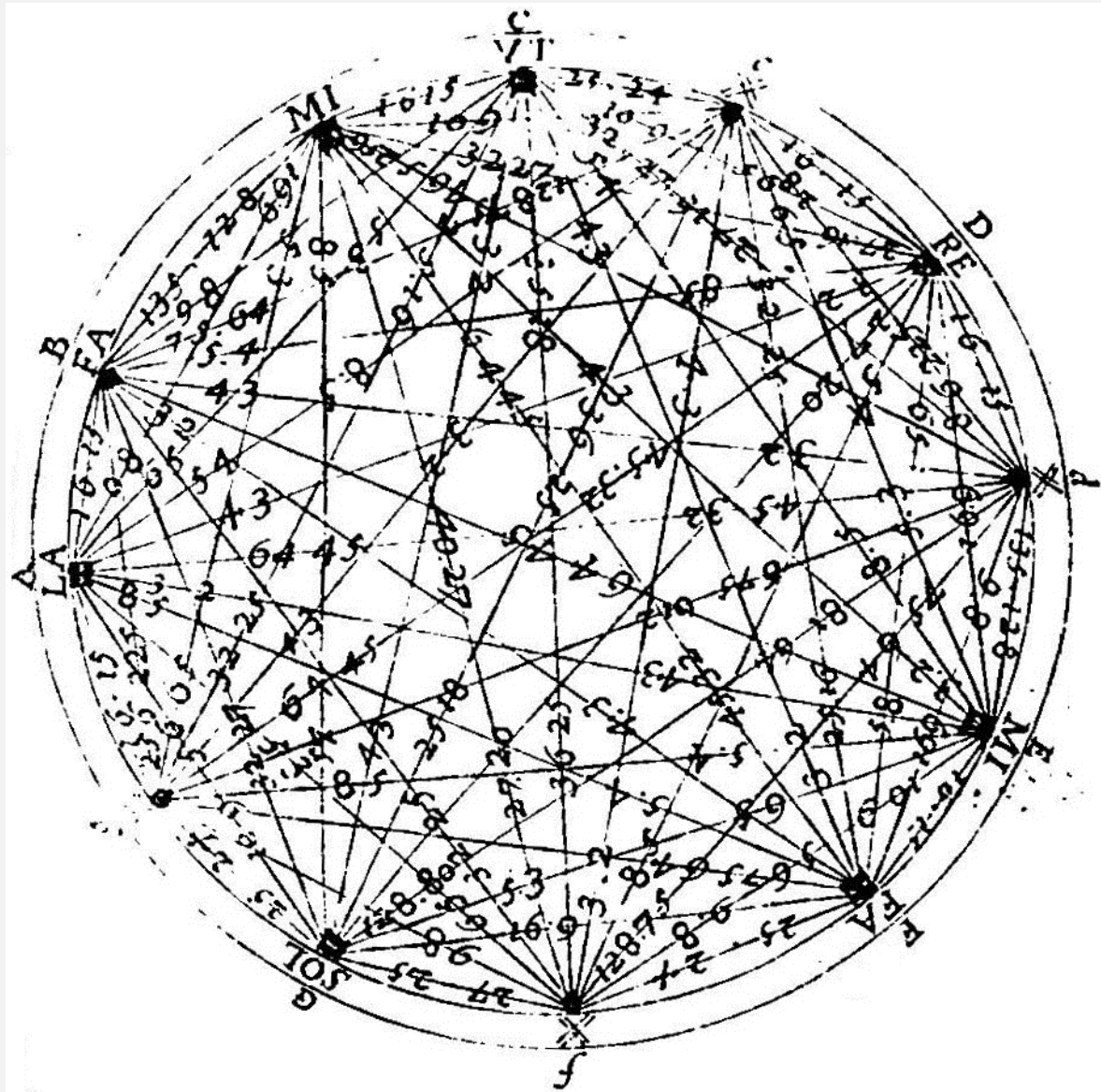
<https://sound-colour-space.zhdk.ch/sets/10012> (copy for Beeckman c. 1628)

<https://sound-colour-space.zhdk.ch/sets/10005> (Leiden 1635)

Marin Mersenne (1636)



Johannes Lippius 1612

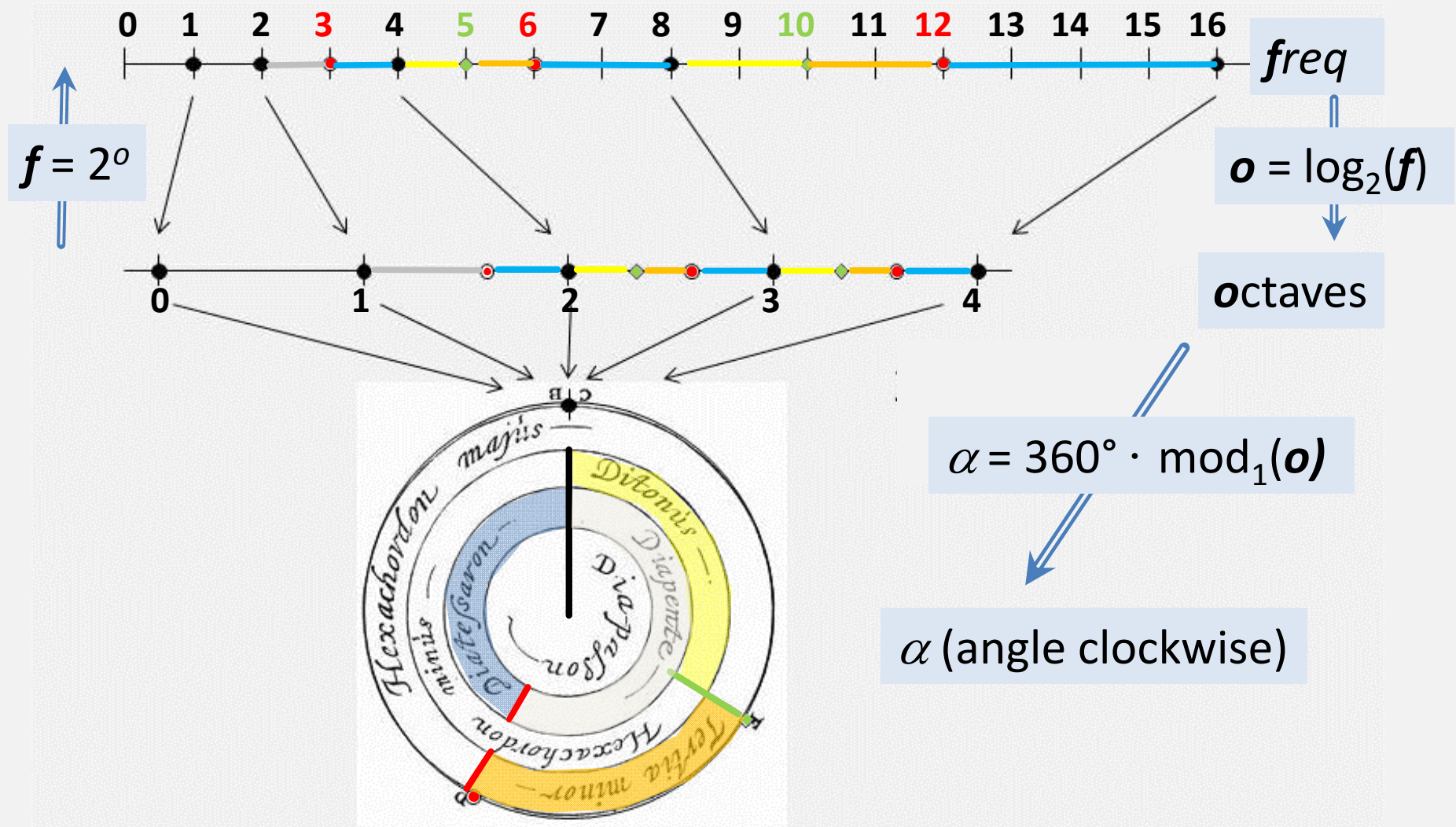


Descartes: Print (1650 / 1653)

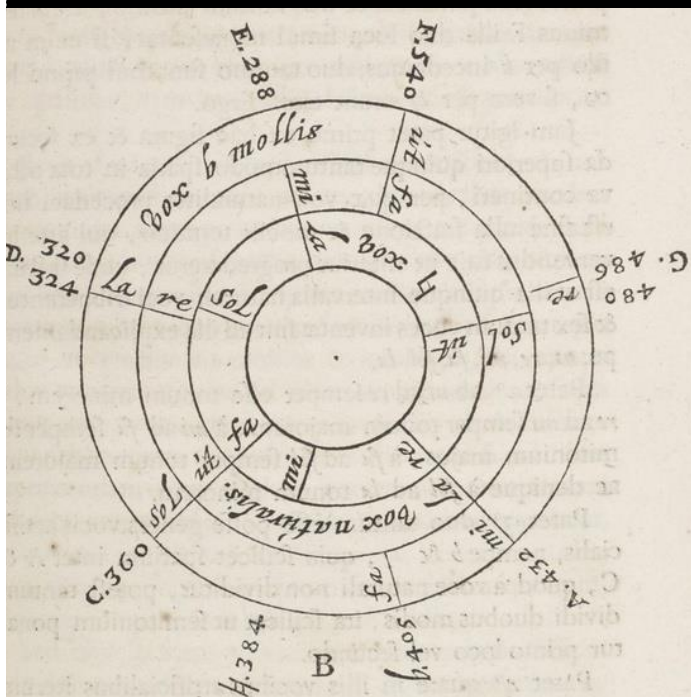
This circular diagram illustrates musical intervals and frequencies. The outer ring contains frequency values: E. 288, F. 540, G. 486, A. 432, B. 405, C. 360, D. 320, and E. 288. The inner rings show intervals: *box b mollis* (top), *box naturalis* (bottom), and *box b mollis* (left). The central circle is divided into segments labeled with musical notes: *La*, *re*, *Sol*, *ut*, *fa*, *mi*, *La*, *re*, *Sol*, *ut*, *fa*, *mi*. The diagram is labeled with 'D' at the top and 'B' at the bottom.

This circular diagram illustrates musical intervals and frequencies. The outer ring contains frequency values: D. 320, E. 288, F. 540, G. 486, A. 432, B. 405, C. 360, and D. 320. The inner rings show intervals: *Third minor*, *Hexachordon minus, or Sixth minor*, *Hexachordon minus, or Sixth minor*, *Diatessaron, or fourth*, *Diatessaron, or fourth*, *Ditone, or third major*, *Third minor*, and *Hexachordon minus, or Sixth minor*. The central circle is divided into segments labeled with musical notes: *La*, *re*, *Sol*, *ut*, *fa*, *mi*, *La*, *re*, *Sol*, *ut*, *fa*, *mi*. The diagram is labeled with 'D' at the top and 'CB' at the bottom.

The maths behind Descartes's diagrams



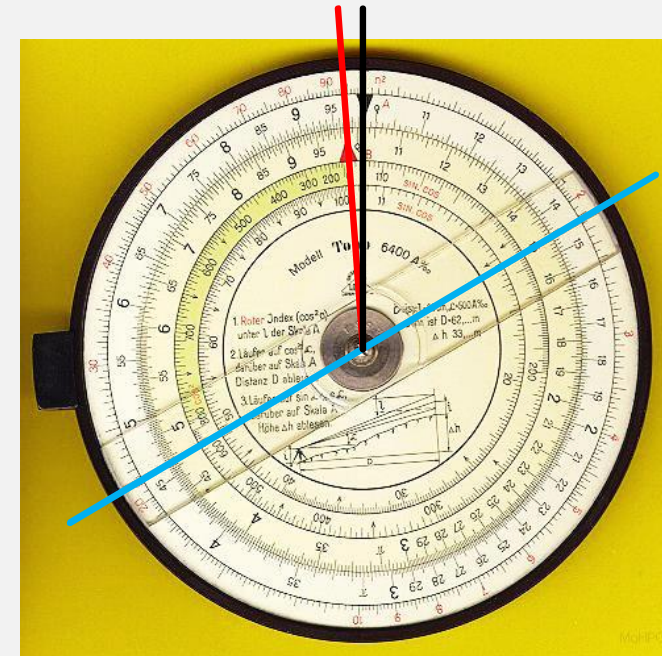
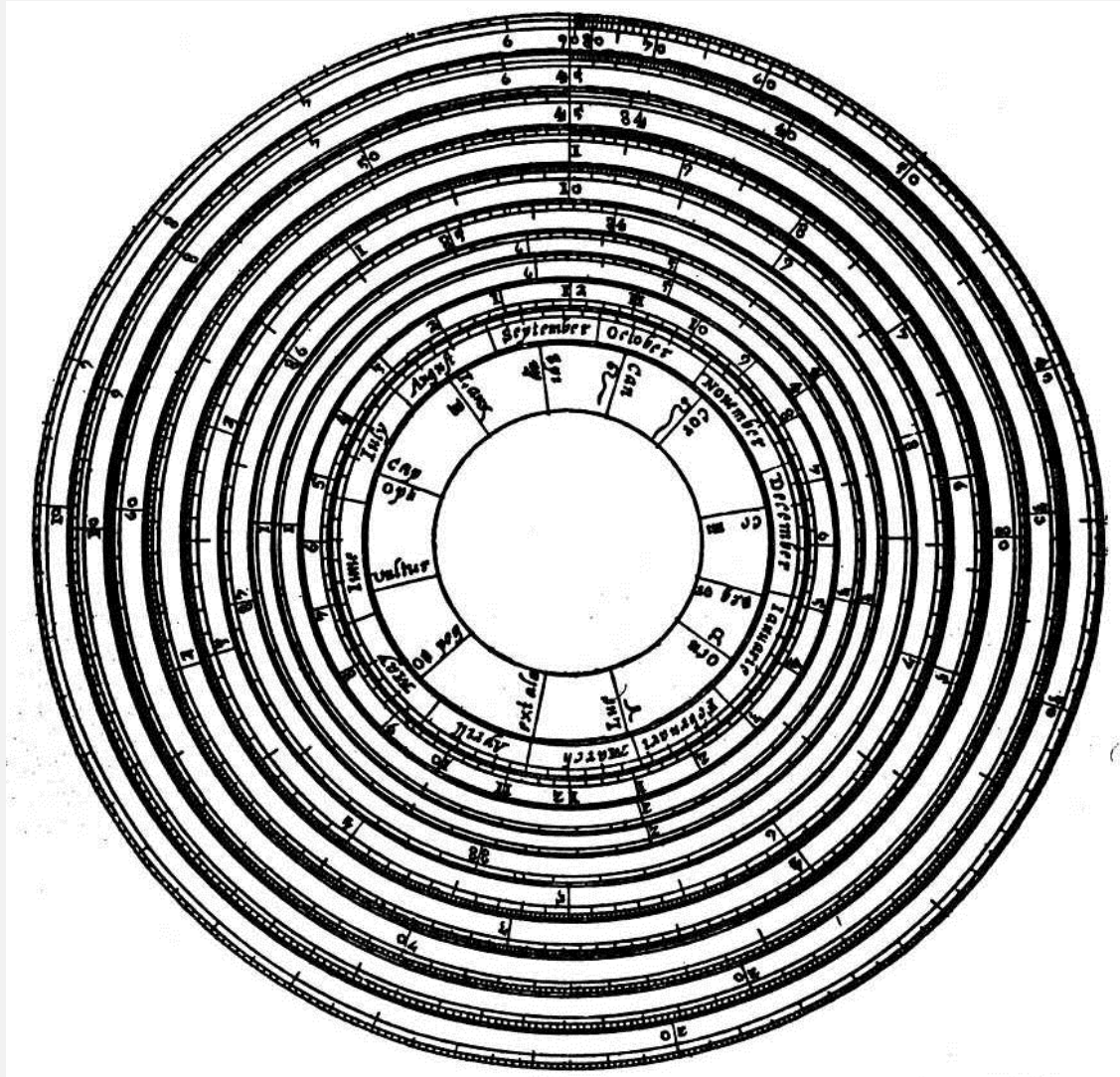
Theinred (12th c.)
Descartes (1650)
Newton (1665)



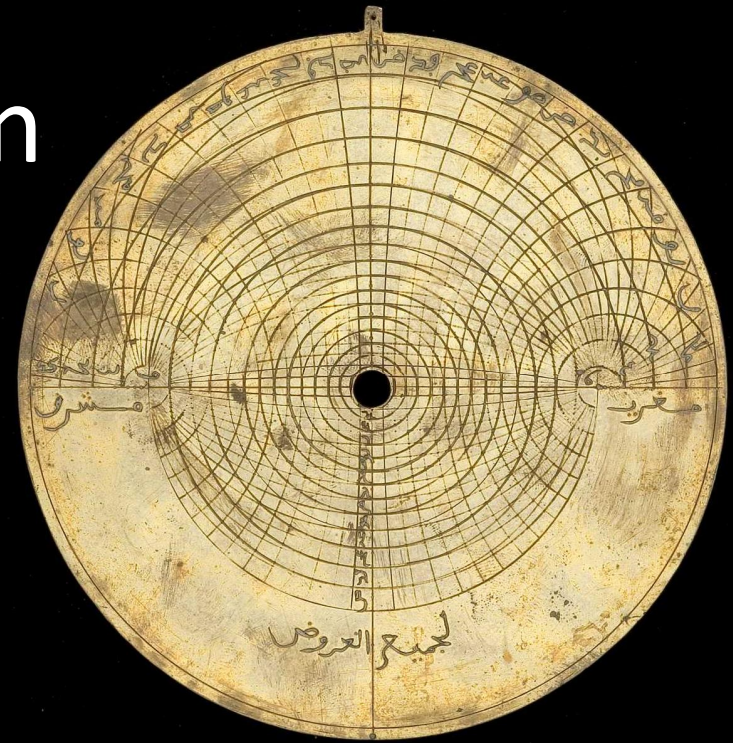
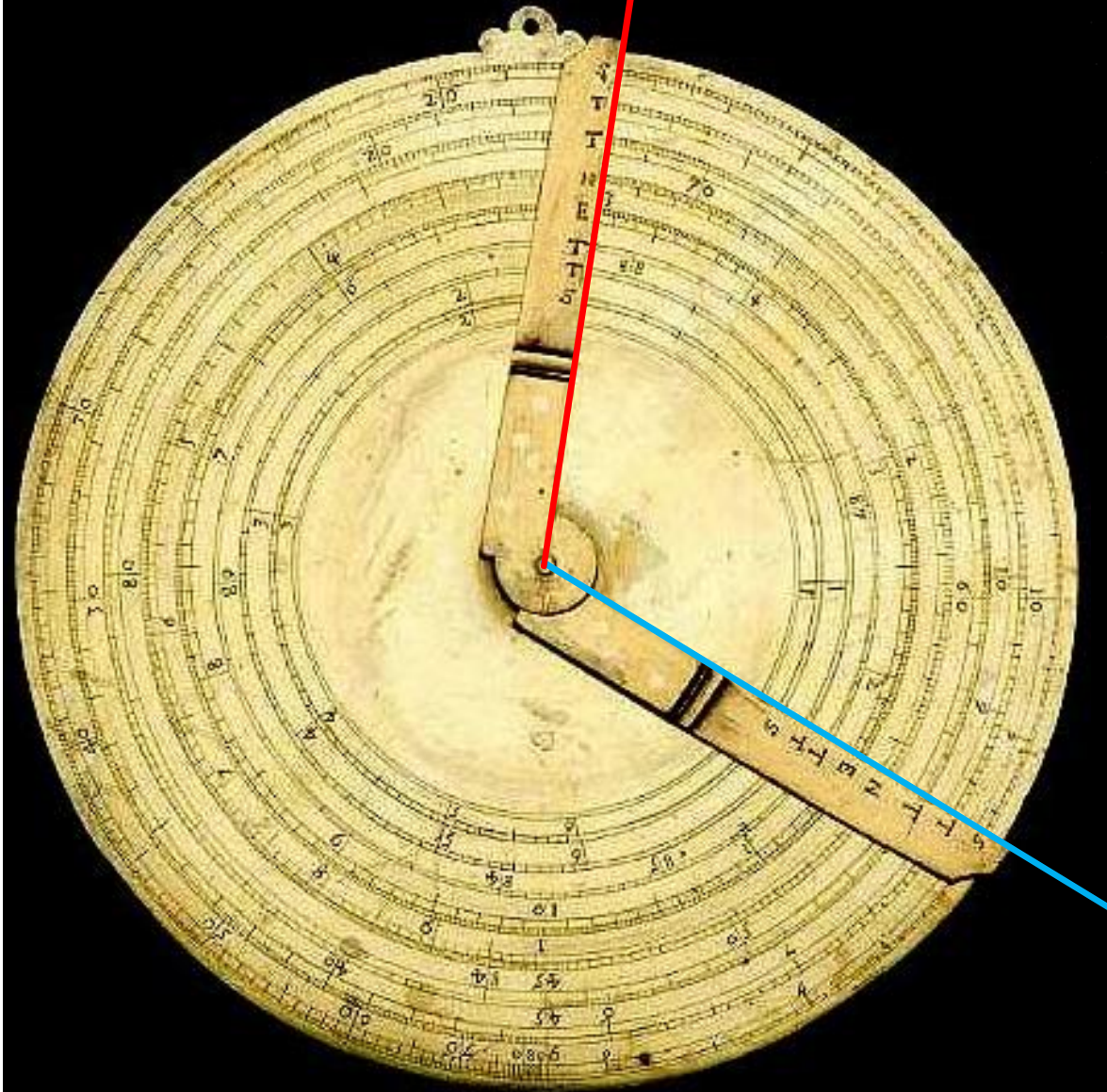
From Bürgi to Oughtred

CIRCULAR SLIDE RULES

William Oughtred (1632)



Elias Allen



Oughtred's circular slide rule with sundial on reverse,

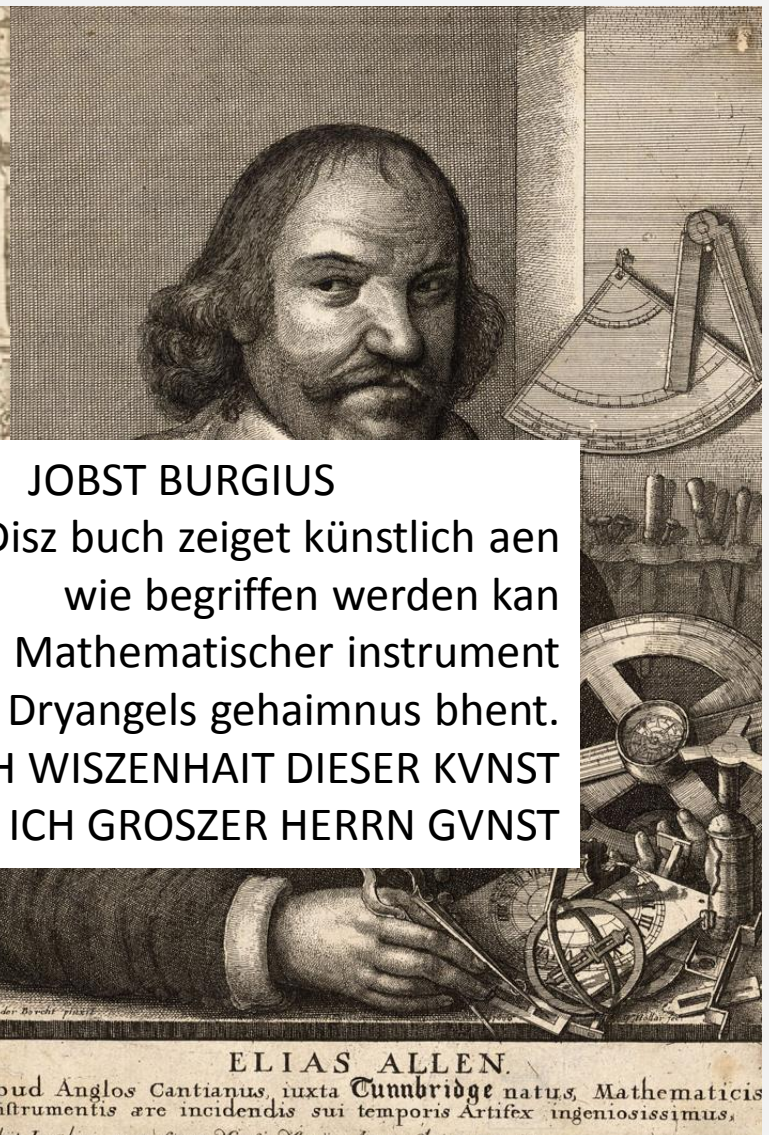


Putnam Gallery Harvard
Elias Allen, London, c. 1633-1640

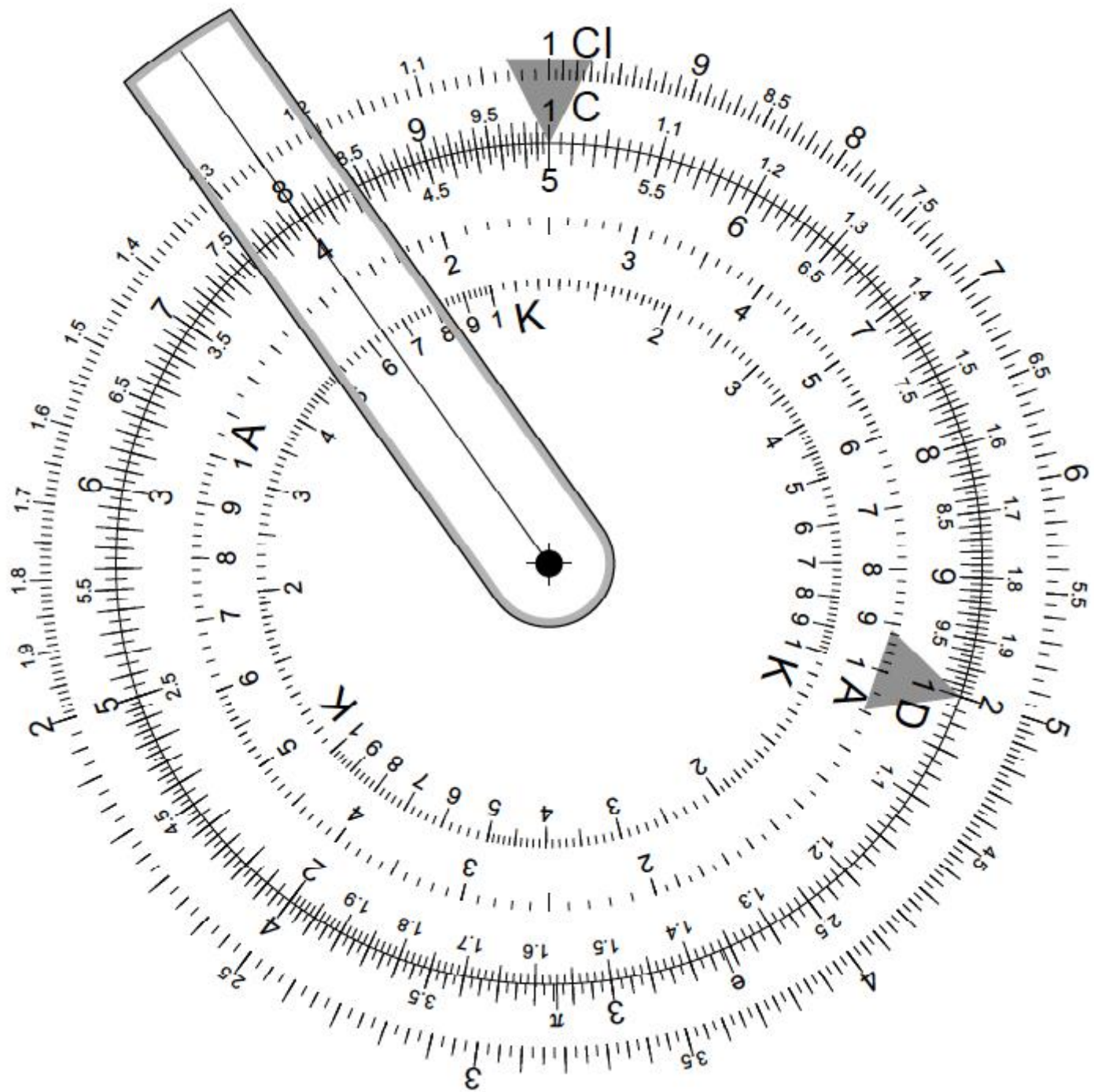
c. 1634 Elias Allen (Oxford)



Oughtred and Allen



JOBST BURGIUS
Disz buch zeigt k^{un}stlich aen
wie begriffen werden kan
Mathematischer instrument
Dryangels gehaimnus bhent.
DVRCH WISZENHAIT DIESER KVNST
ERLANGT ICH GROSZER HERRN GVNST



Octave similarity, geometric sequences of epimoric common factors,
Bürge's base and its musical counterparts

EPIMORIC RATIOS & MUSICAL LOGARITHMS

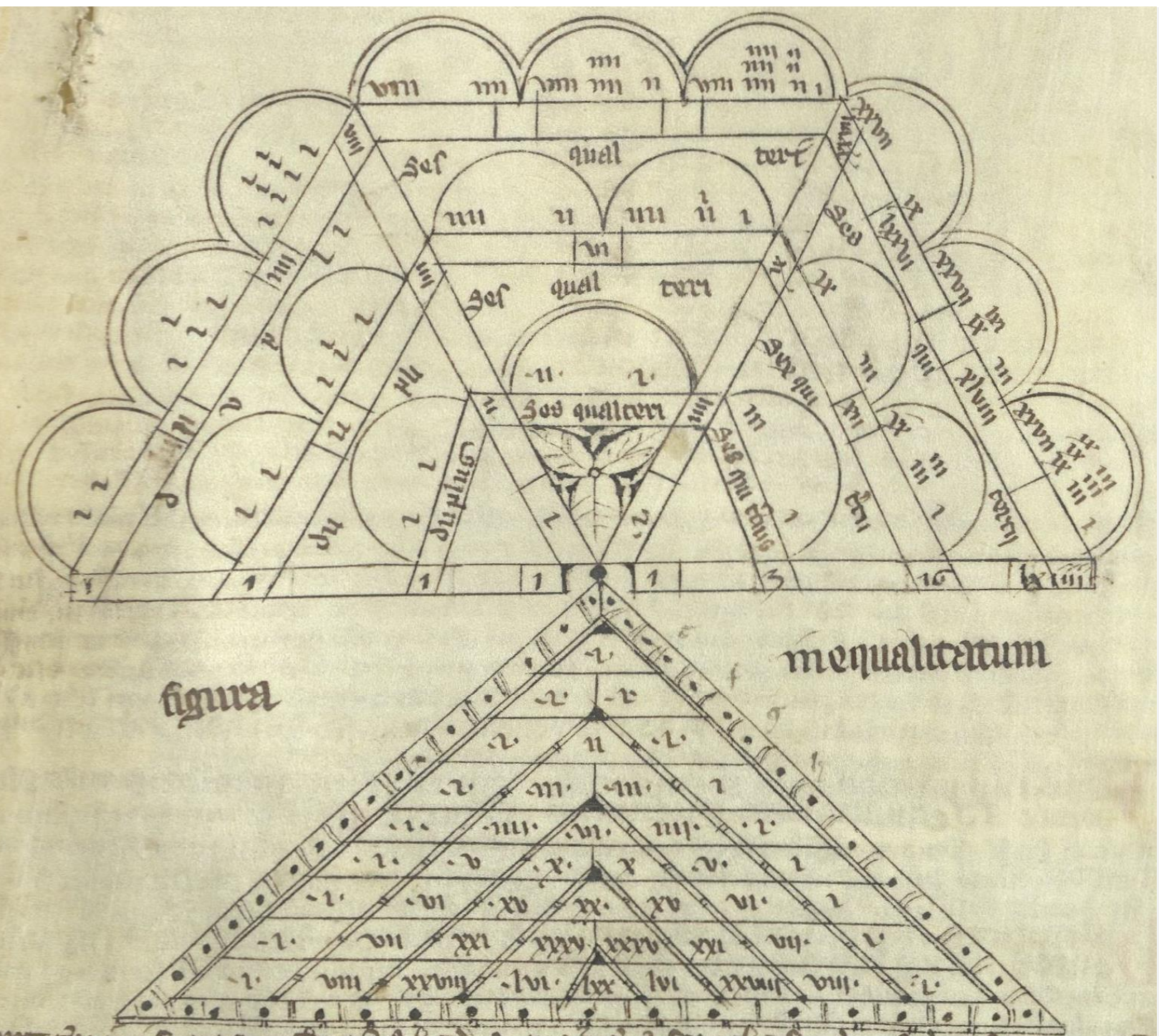
Bürgi's epimoric base

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40	40006	41433	45374
50	50010	51487	55470
60	60015	61543	65584
70	70021	71599	75691
80	80028	81656	85799
90	90036	91714	95907
100	100100045	100601773	101106017
110	10055	11834	16137
120	10066	21895	26239
130	30078	31957	36352
140	40091	42019	46465
150	50105	52084	56580
160	60120	62150	66696
170	70136	72216	76812
180	80153	82283	86930
190	90171	92351	97045
200	100200190	100701410	101202168
210	10210	12491	17289
220	20231	22562	27411
230	30253	32634	37522
240	40276	42707	47657
250	50300	52782	57782
260	60325	62857	67907
270	70351	72932	78032

0	100000000
10	100010000
20	100020001
30	1000300030001
40	10004000600040001
...	
90	100090036.....
100	100100045.....
...	
200	100200190....

Jordanus Nemorarius

Codex Basel F II 33 (mid 14th century)

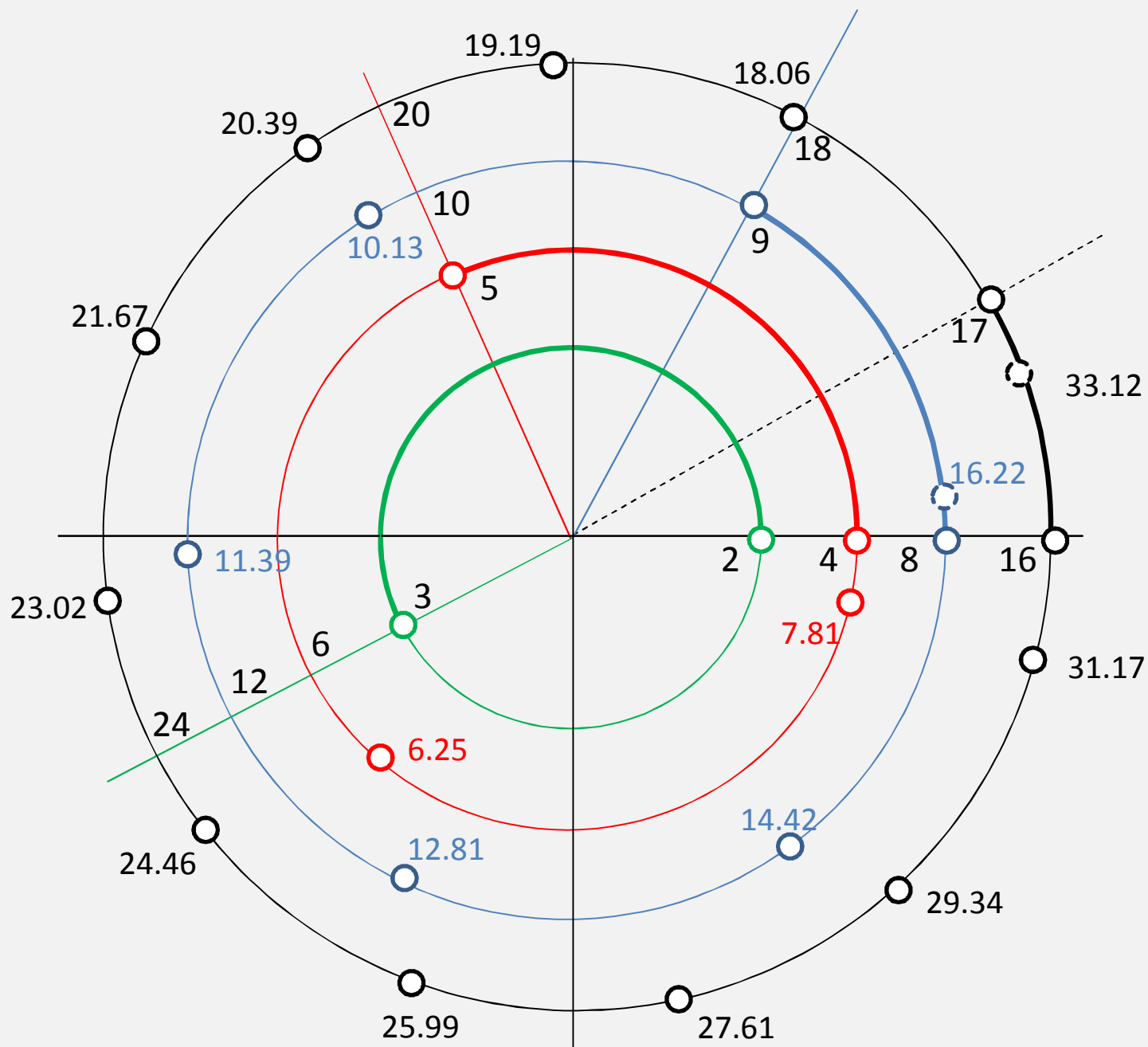


Boethius, De institutione musica (late 10th century)

OC TY PLI

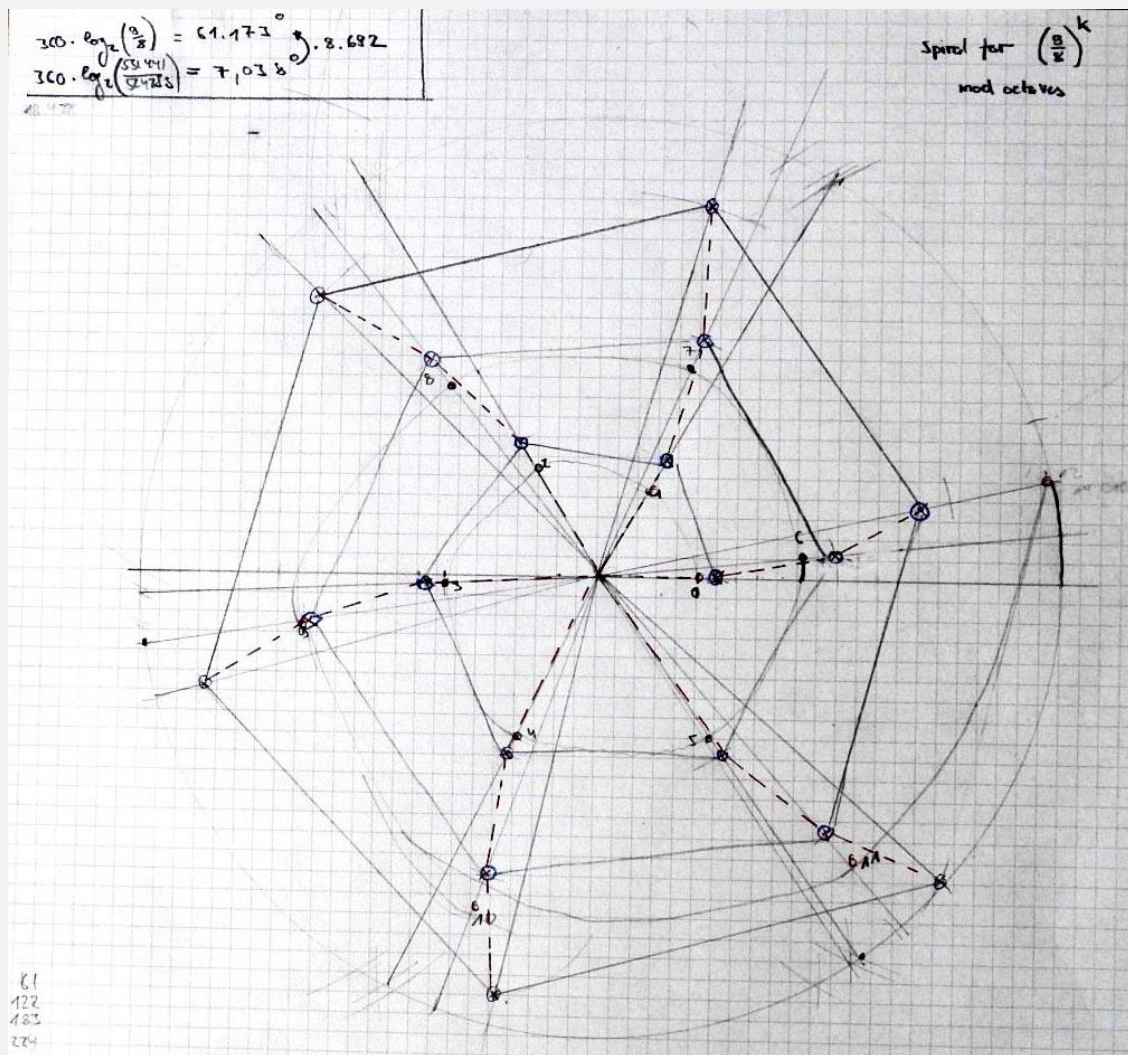
1	viii	lxxiii	dxii	iiii xcv	xxvii dec lx	ceben exlun
	viii	lxxii	dlxxvi	iiii de viii	xxxvi decc lx	cecem decccc
		lxxx	dcxlviii	v lxxxiii	xl i cccc lxx	cecxvii decc lxx
			dcxviii	v decc xxi	xlvi de lvi	ceclxxiii cexl
				vi d lxi	li cccc lxxxv	ccccxviii deccc
					lviii xlvi	cccc lxxii cccc
						o xxxi cccc xli

1	8	64	512	4096	32,768	262,144
	9	72	576	4608	36,864	294,912
		81	648	5184	41,472	331,776
			729	5832	46,656	373,248
				6,561	52,488	419,904
	a	8 a			59,049	472,392
		9 a	+ a			531,441

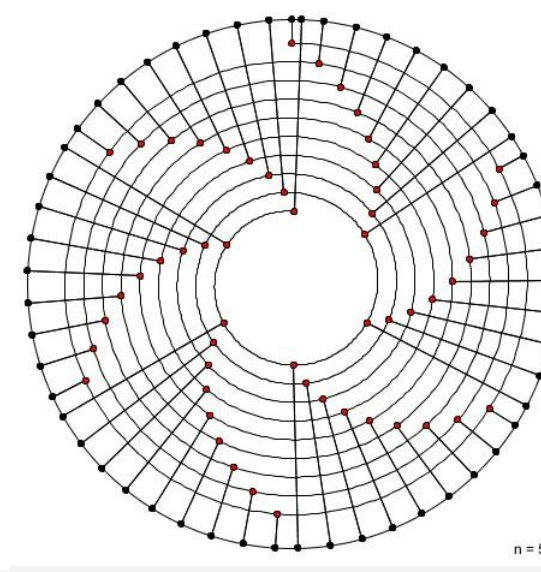
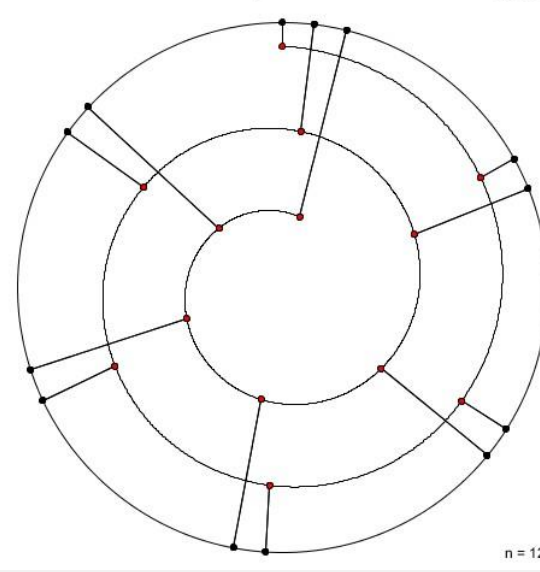
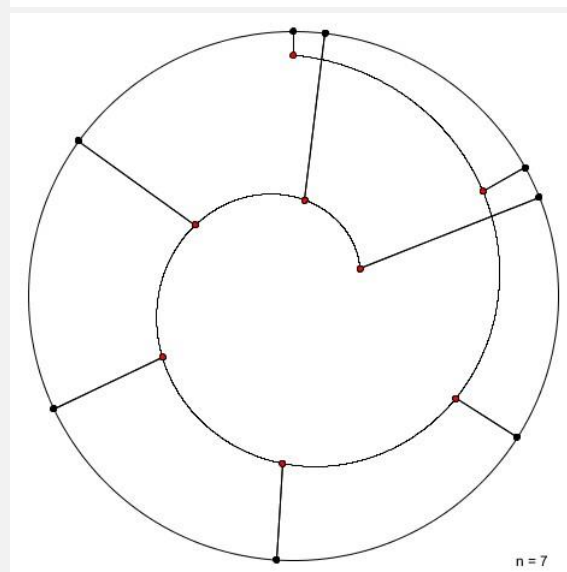
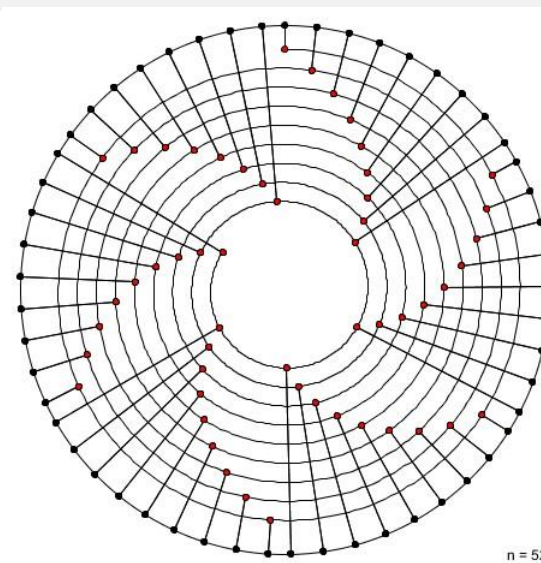
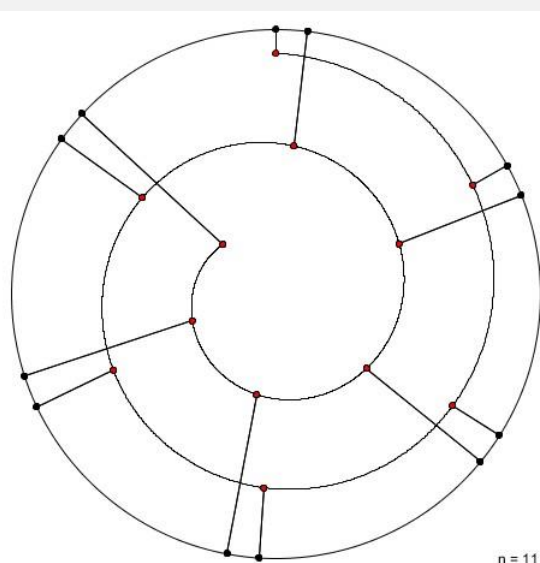
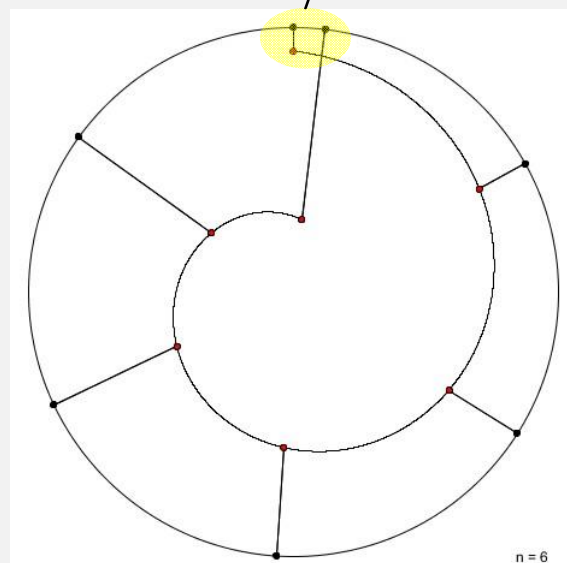


Epimoric sequences within a musical octave

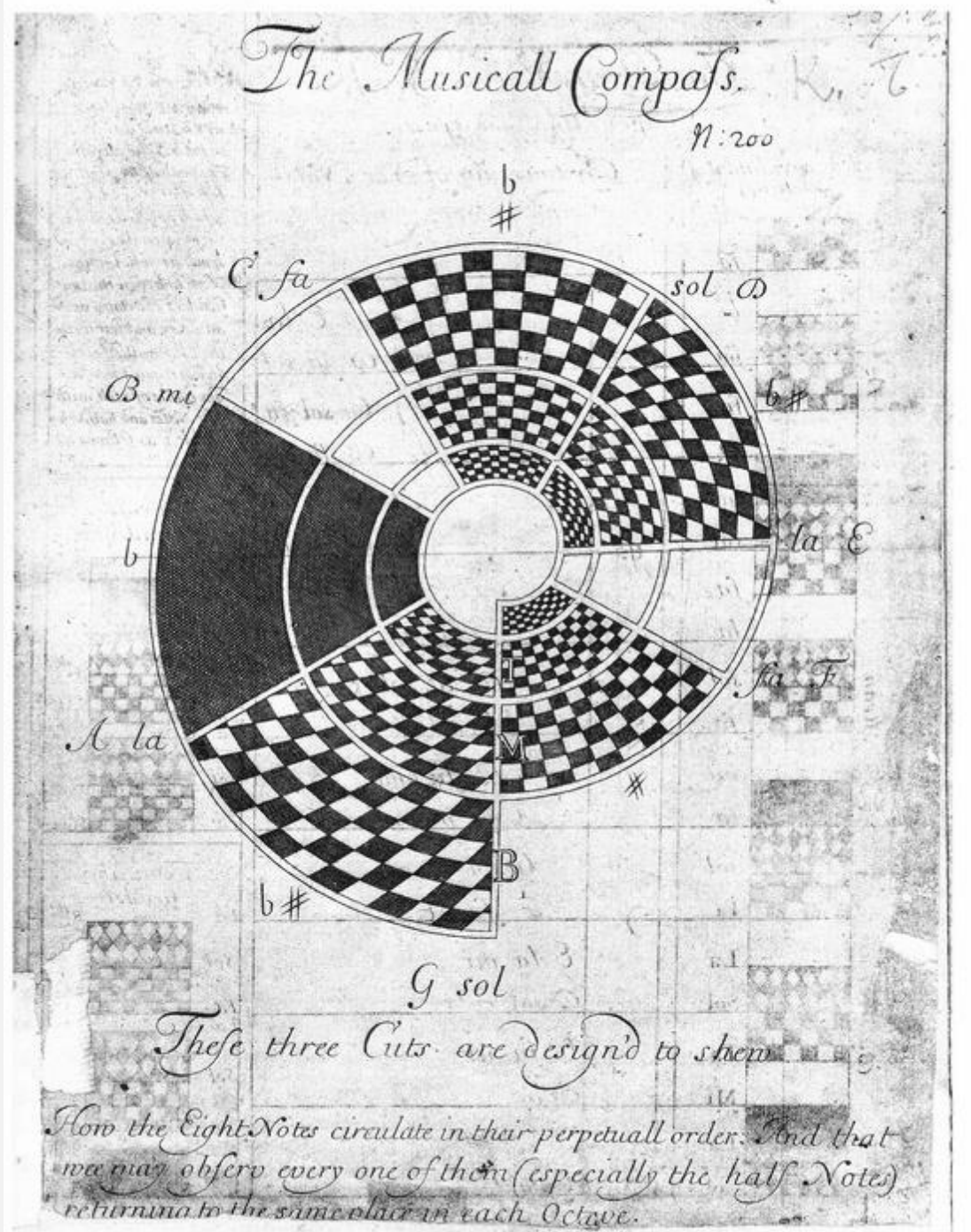
Whole tone spiral: $(9/8)^n$



Pythagorean comma



Whole tone spiral: $(9/8)^n$

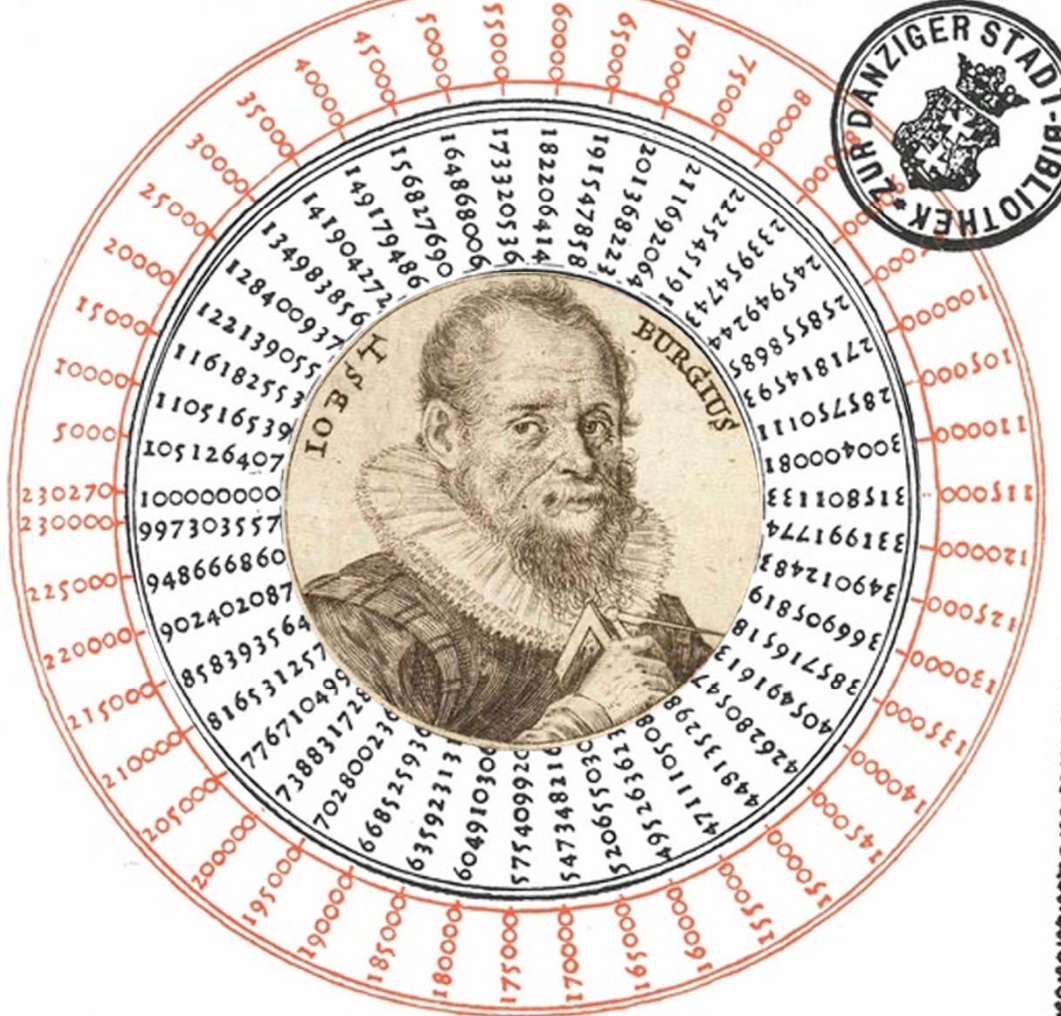


Anonymous 1684
Diatonic spiral

Aritmetische vnd Geometrische Progreß

Tabulen/sambt gründlichem vnterricht/wie solche nützlich
in allerley Rechnungen zugebrauchen/vnd verstanden werden sol.

(Einigen - nicht gedruckten - Unterricht ist ein Manns script beigefügt)



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Selective Bibliography

Brudere, H.: Meilensteine der Rechentechnik, De Gruyter, Oldenburg 2015

Cajori, F.: On the History of Gunter's Scale and the Slide Rule During the Seventeenth Century, Univ. of California Publications in Mathematics, Vol. 1, No. 9, 1920, pp. 187-209

Descartes, René, Compendium Musicae (1618/9). Ms. Copy for Isaac beeckman (c. 1628), Leiden (1635), Groningen (1640). Print: 1650 (Latin), 1653 (English)

Lindley, M.: Stimmung und Temperatur. In: Geschichte der Musiktheorie Bd. 6, Darmstadt, 1987, 109-331

Lippius, Johannes, Synopsis Musicae Novae, Straßburg 1612

Mersenne, Marin: Harmonie universelle. Paris 1636

Muzzulini, D.: Descartes' Töne - Newtons Farben, in: Antonio Baldassarre (hg.), Musik - Raum - Akkord - Bild, Fs. Dorothea Baumann, Peter Lang, Bern 2012, 691-706

-- The Geometry of Musical Logarithms. Acta Musicologica LXXXVII/2 (2015), 193-216

-- Isaac Newton's Microtonal Approach to Just Intonation. Empirical Musicology Review, Vol 15, No 3-4 (2020), pp. 223-248)

Truemper, K.: Die geniale Erfindung der Potenztabelle, Vortrag, Jost Bürgi-Tagung 2021

Voellmy, E.: Jost Bürgi und die Logarithmen. In: Elemente der Mathematik, Beiheft Nr. 5, 1948

Waldvogel, J. : Jost Bürgi and the discovery of the logarithms, Elemente der Mathematik 69, Issue 3, 2014, pp. 89–117

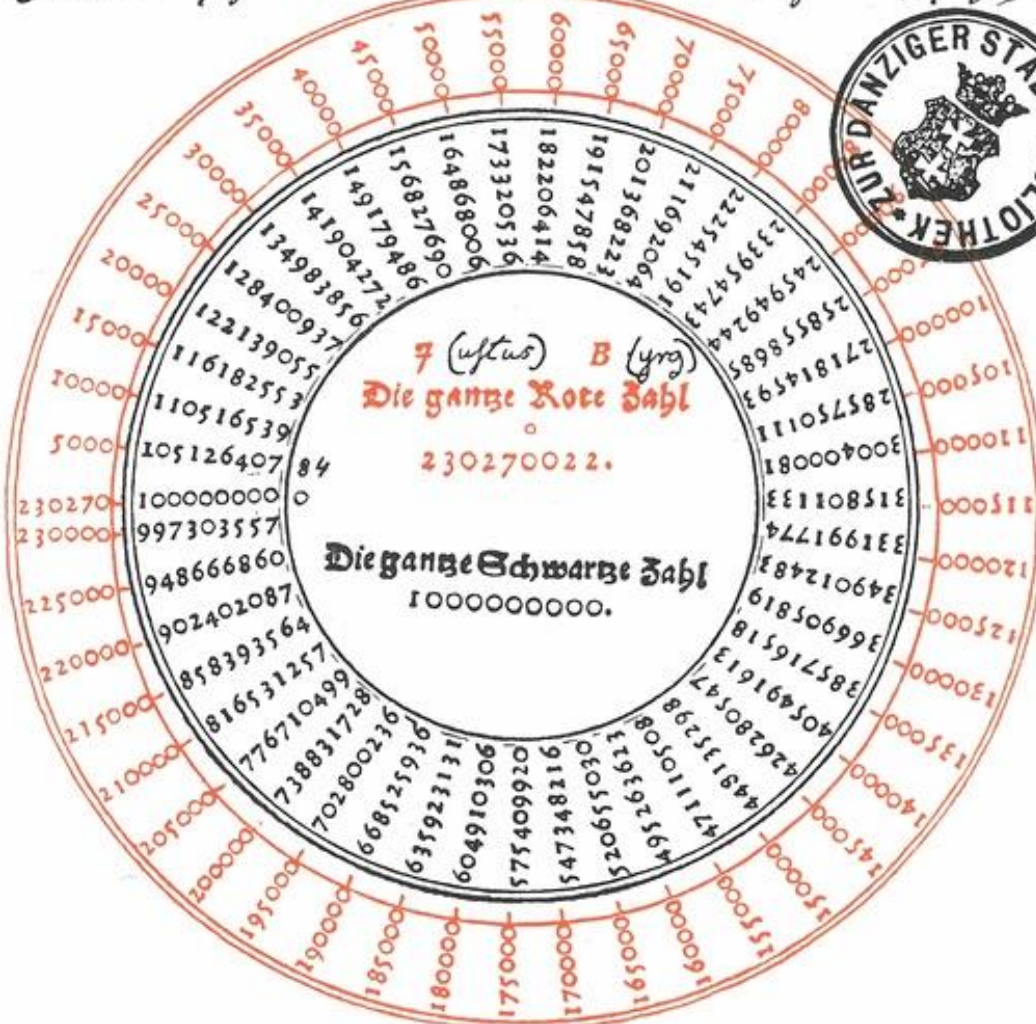
Weiss, S. F.: Ambrosius Wilfflingseder's Erotemata musices, 2019,
<https://www.thinking3d.ac.uk/MusicalVolvellles/> (2019)

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Tabulen/sampt gründlichem vnterricht/wie solche nützlich
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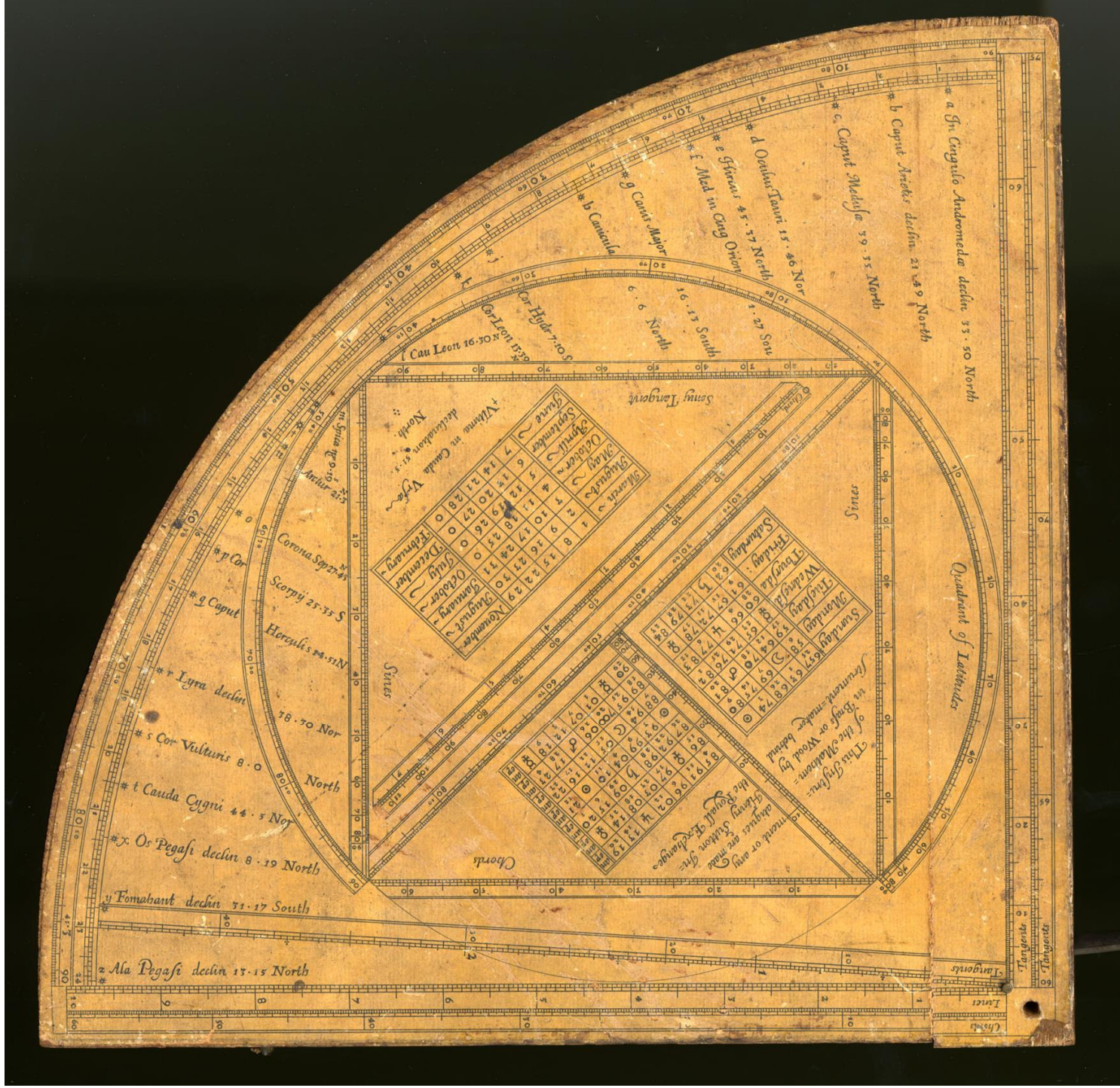


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70	700000000	70071599	70086691
80	800000000	80081656	80096799
90	900000000	90091714	90106907
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110	100100005	100611834	101116137
120	100100066	100621895	101126239
130	100100078	100631957	101136352
140	100100091	100642019	101146465
150	100100105	100652084	101156585
160	100100120	100662150	101166696
170	100100136	100672216	101176812
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Bürgi 1620



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